SOLUTIONS OF PROBABILITY

EXERCISE - 1

PART - I

Section (A) :

A-1. (i) {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(ii) $\{B_1 B_2,\ B_1 B_3,\ B_1 G_1,\ B_1 G_2,\ B_2 B_3,\ B_2 G_1, B_2 G_2,\ B_3 G_1,\ B_3 G_2,\ G_1 G_2\}$

A-2. $P(A) = \frac{3}{11}$; $P(B) = \frac{2}{7}$; P(C) = ?; P (A) + P (B) + P (C) = 1 $\Rightarrow$ P (C) = $\frac{34}{77}$.

A-3. Total number of words formed = $\frac{6!}{3! \times 2!}$ = 60. The number of words containing the pattern

BAN = $\frac{4!}{2!}$ = 12

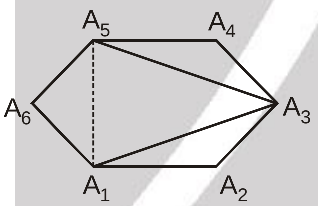
So, the required probability = $\frac{60-12}{60} = \frac{4}{5}$

A-4. Probability = $\frac{5\times4}{9\times8} + \frac{4\times5}{9\times8} = \frac{5}{9}$ (EE or OE)

A-5. A ~ B ~ C; P = $\dfrac{\frac{8!}{3!}}{8!}$

A-6. (i) 6 vertical & 5 horizontal lines.

$p = \frac{5\times4+4\times3+3\times2+2\times1}{{}^6C_2\ \ {}^5C_2} = \frac{4}{15}$



(ii) $A_1A_3A_5$ or $A_2A_4A_6$ = $\frac{2}{{}^6C_3} = \frac{1}{10}$

A-7. **(i)** E_1 : {(1, 1) (1, 2).. (1, 6), (3, 1) … (3, 6), (5,1) (5,6)}
E_2 : {: (2, 6) (3, 6) (4, 4) (5, 3) (6, 2)}

(ii) E_1 : {(4,1) (4,2) ... (4,6)}
E_2 : {(1,5) (2,5) … (6,5)}

A-8. P (Not 8) = 1 – P (8) $\left.\begin{matrix}6+2\\5+3\\4+4\end{matrix}\right\}$ # 5 = 1 – $\frac{5}{36} = \frac{31}{36}$

P (Not 11) = 1 – P (11) 6+5 # 2

= 1 – $\frac{2}{36} = \frac{34}{36}$. Total cases of obtaining 8 or 9 are 7.

Here for cases = 29.

P = $\frac{29}{36}$.

A-9. $P(A) = \frac{5}{10}$; $P(B) = \frac{3}{10}$; $P(C) = \frac{2}{10}$ after the race

$P'(A) = \frac{1}{3}$; $P'(B) + P'(C) = \frac{2}{3}$

That will increase probability of B & C in 3 : 2 respectively

$\therefore \quad P'(B) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

$\therefore \quad P'(C) = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

A-10. Let A(B) be the event that the number on the ticket is divisible by 5(8). Then

A = {5, 10, 15, 20, ..., 95, 100} ; B = {8, 16, 24, 32, ..., 88, 96}

$\Rightarrow \quad A \cap B = \{40, 80\}$; $n(A) = \frac{100}{5} = 20$, $n(B) = 12$, $n(A \cap B) = 2$

The reqd. prob. $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{20}{100} + \frac{12}{100} - \frac{2}{100} = \frac{3}{10}$

A-11. Total number of cases $= {}^{52}C_3 = \frac{52 \times 51 \times 50}{3!} = 22100$

(i) P(all cards of the same suit) = P(all cards are diamonds) + P(all cards are hearts)

+ P(all cards are clubs) + P(all cards are shades) $= 4 \times \frac{{}^{13}C_3}{{}^{52}C_3} = \frac{4 \times 13 \times 12 \times 11}{52 \times 51 \times 50} = \frac{22}{425}$

(ii) P(A king, a queen, a jack) = P(a king) × P(a queen) × P(a jack) $\Rightarrow \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_3} = \frac{16}{5525}$

A-12. $A \rightarrow$ sum is 8, $B \rightarrow$ sum is 11

If A occurs naturally B is not allowed so 'A total of 8 but not 11' is equivalent to sum of '8' is obtained now n(S) = 6 × 6

n(E) = {(2, 6), (6, 2), (4, 4), (3, 5), (5, 3)} $\Rightarrow$ P = 5/36

A-13. n (S) = total number of arrangements = 12!; n (E) = alternate arrangement = 2.6! 6! $P = \frac{2.6!.6!}{12!}$

A-14. (i) total ways of drawing 4 cards $= {}^{52}C_4$. one card each from each suit $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$P = \frac{13 \times 13 \times 13 \times 13}{{}^{52}C_4}$

(ii) Value of a card voices like 2 , 3 .... 10, J, O, K , A i.e., 13 values are possible of which 4 different can be selected as ${}^{13}C_4$:

also any particular value is available in four suits $\Rightarrow n(E) = {}^{13}C_4\ 4^4$

$P = \frac{{}^{13}C_4 \quad 4^4}{{}^{52}C_4}$

Section (B) :

B-1. We known that

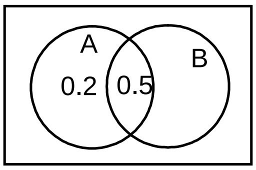
$n(A - B) = n(A) - n(A \cap B) = n(A \cup B) - n(B) = n(A \cap \bar{B}) = n(S) - n(\bar{A} \cap B)$

dividing by n(S) in total we get

$P(A - B) = P(A) - P(A \cap B) = P(A \cup B) - P(B) = P(A \cap \bar{B}) = 1 - P(\bar{A} \cap B)$

B-2 (i) $P(A - \bar{B}) = P(A \cap B) = 0.5$ (ii) $P(\bar{A} \cup B)$

$= 1 - P\left(\overline{\bar{A} \cup B}\right) = 1 - \{P(A \cap \bar{B})\} = 1 - \{P(A) - P(A \cap B)\} = 1 - P(A) + P(A \cap B) = 1 - 0.7 + 0.5 = 0.3 + 0.5 = 0.8$



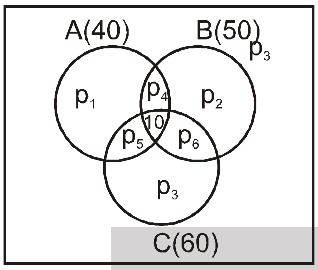
B-3. (i) $p(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.48 - 0.16 = 0.72$

(ii) $P(B) - P(A \cap B) = 0.32$

(iii) $p\left(\bar{A} \cap \bar{B}\right) = p(A \cup B)^c = 1 - p(A \cup B) = 1 - 0.72 = 0.28$

insert values and obtain answer (iv) $P(A \cup B) - P(A \cap B) = 0.56$

B-4.



$p_1 + p_2 + p_3 = 70$ and $p_4 + p_5 + p_6 = 25$

A : Event that he has membership of exactly two clubs $P(A) = \frac{25}{70+25+10} = \frac{25}{115} = \frac{5}{21}$

Section (C) :

C-1. (i) $B_1 \rightarrow$ boy (given)

Possible scenario (Ist place $\rightarrow$ elder
IInd place $\rightarrow$ younger)

B_1G_1 $\quad G_1B_1$ $\quad B_1B_2$

Other child is girl $= \frac{2}{3}$

C-2. Score less than 5 means the occurrence of 1, 2, 3, or 4. Now on the last throw we should not obtain a score less then 2 i.e. one. Clearly the favourable outcomes are 2, 3 or 4.

Thus the required probability $= \frac{3}{4}$

C-3. (i) 8/52 $\qquad$ (ii) $\frac{13+4-1}{52} = \frac{4}{13}$

C-4. $P(E_1) = 2/7$ $\qquad P(E_2) = 6/11$; $P(E_1UE_2) = P(E_1) + P(E_2) - P(E_1)\,P(E_2) = \frac{2}{7} + \frac{6}{11} - \frac{2}{7} \times \frac{6}{11} = \frac{52}{77}$

C-5. $p(A) = \frac{13}{52} + \left(\frac{39}{52}\right)^3 \left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^6 \left(\frac{13}{52}\right) + \ldots. = \frac{13/52}{1-\left(\frac{39}{52}\right)^3} = \frac{16}{37}$

$p(B) = \left(\frac{39}{52}\right)\frac{13}{52} + \left(\frac{39}{52}\right)^4 \frac{13}{52} + \left(\frac{39}{52}\right)^7 \left(\frac{13}{52}\right) + \ldots = \frac{3/16}{1-\left(\frac{3}{4}\right)^3} = \frac{12}{37}$

$p(C) = \left(\frac{39}{52}\right)^2 \left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^5 \left(\frac{13}{52}\right) + \ldots = \frac{9/64}{1-\left(\frac{3}{4}\right)^3} = \frac{9}{37}$

C-6. here $\quad$ x : C & D $\quad$ separated
y : A & B $\quad$ together

$P(x/y) = \frac{P(x \cap y)}{P(y)} = \frac{3! \times 2! \times {}^4C_2 \times 2!}{5!\,2!} = \frac{3}{5}$

C-7. $XI \rightarrow 5/50$

$XII \rightarrow 8/50$ $\qquad P(XI) = \frac{2}{5}$, $P(XII) = \frac{3}{5}$

$P(\text{Brilliant}) = \frac{2}{5} \times \frac{1}{10} + \frac{3}{5} \times \frac{8}{50} = \frac{17}{125}$

C-8. B1 → 5R + 2B; B2 → 2R + 6B

(i) $P(R) = \frac{1}{2} \times \frac{5}{7} + \frac{1}{2} \times \frac{2}{8} = \frac{27}{56}$

(ii) A : Ball drawn is blue

B_1 : From B_1

B_2 : From B_2

$$P(B_1/A) = \frac{P(A/B_1)\ \ P(B_1)}{P(A/B_1)\ \ P(B_1) + P(A/B_2)\ \ P(B_2)} = \frac{\frac{2}{7} \times \frac{1}{2}}{\frac{2}{7} \times \frac{1}{2} + \frac{6}{8} \times \frac{1}{2}} = \frac{8}{29}$$

C-9. No of kings left are 3. cards are 51; $p = \frac{3}{51} = \frac{1}{17}$

C-10. Here S = {1, 2, 3 . . . . . 12}. Total events n(s) = 12.

Let E : number on the drawn card is more than 3

F : number on the card is even number

E = {4, 5, 6, 7, 8, 9, 10, 11, 12}

F = {2, 4, 6, 8, 10, 12}

$E \cap F$ = {4, 6, 8, 10, 12}

$$P(E) = \frac{9}{12}\ ,\ P(F) = \frac{6}{12}\ ,\ P(E \cap F) = \frac{5}{12} \quad \Rightarrow P\left(\frac{F}{E}\right) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{5}{12}}{\frac{9}{12}} = \frac{5}{9}$$

C-11. M : event that all the materials will be delivered at the correct time.

F : event that the building programme will be completed on time.

$$P\left(\frac{F}{M'}\right) = \frac{P(F \cap M')}{P(M')} = \frac{P(F) - P(F \cap M)}{1 - P(M)} = \frac{0.7 - 0.65}{1 - 0.8} = \frac{1}{4}$$

Section (D) :

D-1. $P = {}^{14}C_{13}\ \frac{1}{2}\left(\frac{1}{2}\right)^{14-13} = 14 \times \frac{1}{2^{13}}\ \frac{1}{2} = \frac{7}{2^{13}}$

D-2. In a question of given type probability of giving correct answer $= \frac{1}{15}$

Exactly two correct answers $\equiv {}^{10}C_2 \times \left(\frac{1}{15}\right)^2 \times \left(\frac{14}{15}\right)^8$

D-3. Total cards = 52

Spade cards = 13

Probability of success $p = \frac{13}{52} = \frac{1}{4}$

∴ $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Let X be the number of success

∴ $P(X = 0) = q^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64} \Rightarrow P(X = 1) = 3\,q^2 P = 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{27}{64}$

$P(x = 2) = 3qp^2 = 3\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 = \frac{9}{64} \Rightarrow P(X = 3) = P^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

∴ Required probability distribution is

X	0	1	2	3
P(X)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

D-4. $E_B = E_C = \frac{1}{5} \times 100 + \frac{4}{5} \times \frac{10}{100} \times 100 = 28$

D-5. Box = 2 R + 3B. p (Rad, blue) = $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10} = p$

x	0	1	2	3
p	$^3C_0p^0(1-p)^3$	$^3C_1p(1-p)^2$	$^3C_2p^2(1-p)$	$^3C_3p^3$

=

x_i	0	1	2	3
p_i	$\left(\frac{19}{25}\right)^3$	$18 \times \frac{19^2}{25^3}$	$108 \times \frac{19}{25^3}$	$\frac{216}{25^3}$

D-6. Distribution

x	0	1	2	3	4	5
p	$^5C_0\left(\frac{1}{2}\right)^5$	$^5C_1\left(\frac{1}{2}\right)^5$	$^5C_2\left(\frac{1}{2}\right)^5$	$^5C_3\left(\frac{1}{2}\right)^5$	$^5C_4\left(\frac{1}{2}\right)^5$	$^5C_5\left(\frac{1}{2}\right)^5$

mean = np = $5 \times \frac{1}{2}$ = 2.5; variance = npq = $5 \times \frac{1}{2} \times \frac{1}{2} = 1.25$

PART - II

Section (A) :

A-1. P (A) = $\frac{13}{52}$ – spade $\Rightarrow$ P(A) = $\frac{1}{4}$; P (B) = $\frac{4}{52}$ Ace $\Rightarrow$ P (B) = $\frac{1}{13}$

They are independent event As $P(A \cap B) = P(A).P(B) = 1/52$

A-2. Since sum of 1+12+3 +......9 = $\frac{9 \times 10}{2}$ = 45 is disisible by 9, hence all number will be divisible by 9.

A-3. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 0 \quad \Rightarrow$ where ad = 1, bc = 1 or ad = – 1, bc = – 1

which occur in eight ways. Total number of 2 × 2 determinants from {–1, 1} is 16.

Thus required probability is $\frac{8}{16} = \frac{1}{2}$

A-4. According to the given condition p = p(E) = $\frac{1}{2}$, q = $\frac{1}{2}$

$^nC_3\left(\frac{1}{2}\right)^{n-3}\left(\frac{1}{2}\right)^3 = {}^nC_4\left(\frac{1}{2}\right)^{n-4}\left(\frac{1}{2}\right)^4$, where n is the number of times dice is thrown

$\Rightarrow$ $^nC_3 = {}^nC_4 \Rightarrow n = 7$. Thus required probability = $^7C_1\left(\frac{1}{2}\right)^7 = \frac{7}{2^7} = \frac{7}{128}$

A-5. B can obtain number > 9 in these manner ($\because$ A and B are independent events)

(5,5), (6,5), (5,6), (6,6), (6,4), (4,6) $\Rightarrow$ P = $\frac{6}{36} = \frac{1}{6}$

A-6. Roots of the equation $x^2 + qx + \frac{3q}{4} + 1 = 0$ are real if $\Delta = q^2 - 4\left(\frac{3q}{4} + 1\right) \geq 0$

$\Rightarrow$ $q^2 - 3q - 4 \geq 0 \Rightarrow (q + 1)(q - 4) \geq 0 \Rightarrow q \leq -1$ or $q \geq 4$.

$\Rightarrow$ possible value of q are 4, 5, 6, 7, 8, 9, 10, –10, –9, –8, –7, –6, –5, –4, –3, –2, –1.

$\Rightarrow$ probability = $\frac{17}{21}$.

A-7. So $\dfrac{4.{}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$

A-8. Total cases x^8 : $(x^0 + x^1 + ...x^6)^4 = \left(\dfrac{1-x^7}{1-x}\right)^4 = (1 - x^7)^4 (1 - x)^{-4} = (1 - 2x^7)^2 (1 - x)^{-4} = (1 - 4x^7)(1 - x)^{-4}$

Total ways = 7^4. Favourable ways = ${}^{4+8-1}C_8 - 4.{}^{4+1-1}C_1 = {}^{11}C_8 - 4 \times 4 = 165 - 16 = 149$. P = $\dfrac{149}{7^4}$

A-9. $1 - P(BB)$; $1 - 1/2 \times 1/2 = 1 - 1/4 = 3/4$

Section (B) :

B-1. There are 4 possible cases for an elements

(i) neither present in A nor in B
(ii) present both in A and B
(iii) present in A and absent in B
(iv) present in B and absent in A

Case (iii) and (iv) are favorable = $\left(\dfrac{2}{4}\right)^n = \dfrac{1}{2^n}$

B-2. (i) $\because$ $0 \le P(A \cap B) \le \min(P(A), P(B))$ ......(i)

and $0 \le P(A \cup B) \le 1$

So $P(A) + P(B) - 1 \le P(A \cap B) \le P(A) + P(B)$ ......(ii)

(i) $\cap$ (ii)

$P(A) + P(B) - 1 \le P(A \cap B) \le \min(P(A), P(B))$

(ii) $\because$ $\dfrac{4}{15} \le P(A \cap B) \le \dfrac{3}{5} \Rightarrow \dfrac{4}{15} \le P(A) + P(B) - P(A \cup B) \le \dfrac{3}{5}$

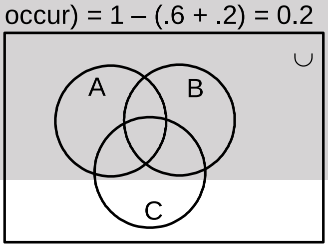
$P(A) + P(B) - \dfrac{3}{5} \le P(A \cup B) \le P(A) + P(B) - \dfrac{4}{15} \Rightarrow \dfrac{2}{3} \le P(A \cup B) \le 1$

(iii) $P(A \cap B') = P(A) - P(A \cap B)$

$\because$ $P(A \cap B) \in \left[\dfrac{4}{15}, \dfrac{3}{5}\right] \Rightarrow 0 \le P(A \cap B') \le \dfrac{1}{3}$

B-3 $\dfrac{{}^{10}C_2\, 3^8}{4^{10}} = 5.\left(\dfrac{3}{4}\right)^{10}$

B-4 P(at least two of A, B, C occur) = 1 – (.6 + .2) = 0.2



Section (C) :

C-1. A $\to$ first critic reviews favourbly P (A) = $\dfrac{5}{7}$; B $\to$ second critic reviews favourbly P (B) = $\dfrac{4}{7}$

C $\to$ third critic reviews favourbly P (B) = $\dfrac{3}{7}$

For majority $P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + (\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= \dfrac{5}{7} \times \dfrac{4}{7} \times \dfrac{4}{7} + \dfrac{5}{7} \times \dfrac{3}{7} \times \dfrac{3}{7} + \dfrac{2}{7} \times \dfrac{4}{7} \times \dfrac{3}{7} + \dfrac{5}{7} \times \dfrac{4}{7} \times \dfrac{3}{7} = 209/343$

C-2. p (A) = $\frac{1\times 6}{36} = \frac{1}{36}$, p (B) = $\frac{6}{36} = \frac{1}{6}$ $\left.\begin{matrix} 6+1 \\ 5+2 \\ 4+3 \end{matrix}\right\}$

A $\cap$ B = $\frac{1}{36}$; p (A $\cap$ B) = p (A) × p (B)

C-3. Total events = 6 × 6 = 36. A = Getting the number 5 at least once

$\Rightarrow$ A = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)}

B = Getting the sum of numbers to be 8. $\Rightarrow$ B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$\Rightarrow$ A $\cap$ B = {(3, 5), (5, 3)}

$\therefore$ P(A) = $\frac{11}{36}$; P(B) = $\frac{5}{36}$; P(A $\cap$ B) = $\frac{2}{36}$

Now, the Reqd. Prob.= P(A/B) = $\frac{P(A\cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$

C-4. Fails : $A \quad \bar{B} + \bar{A} \quad B + \bar{A} \quad \bar{B}$

Probility $\frac{\bar{A} \quad B}{A \quad \bar{B} + \bar{A} \quad B + \bar{A} \quad \bar{B}} = \frac{(.1) \quad (.8)}{(.1)(.8) + (.9)(.2) + (.1)(.2)} = \frac{8}{8+18+2} = \frac{2}{7}$

C-5. A $\rightarrow$ missing card is red P (A) = $\frac{1}{2}$; B $\rightarrow$ missing card non red P (B) = $\frac{1}{2}$ E $\rightarrow$ card drawn is red

P(E) = P(A) P(E/A) + P(B) P(E/B) = $\frac{1}{2} \times \frac{25}{51} + \frac{1}{2} \times \frac{26}{51} = \frac{1}{2} \Rightarrow$ P(A/E) = $\frac{P(E/A) \quad P(A)}{P \quad (E)} = \frac{\frac{25}{51}\times\frac{1}{2}}{\frac{1}{2}} = \frac{25}{51}$

C-6. Here A: Forget to water; B: Withered

P(A/B) = $\frac{P(A) \quad . \quad P(B/A)}{\Sigma \quad P(A) \quad . \quad P(B/A)} = \frac{\frac{2}{3} \quad . \quad \frac{3}{4}}{\frac{2}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}} = \frac{6}{6+2} = \frac{3}{4}$

C-7. p (A / B) = $\frac{p(A\cap B)}{p(B)} = \frac{0.1+0.1}{0.3} = \frac{2}{3}$. similarly evaluate others

Section (D) :

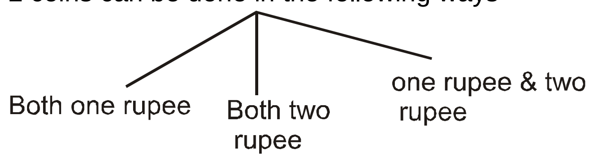
D-1. 2W & 4B $\Rightarrow$ P = ${}^5C_4 \times \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^1 + {}^5C_5 \left(\frac{2}{6}\right)^5$

D-2. ${}^3C_2 P^2$ (1–P) = 12 ${}^3C_3$ P^3 $\Rightarrow$ 1 – P = 4 P $\Rightarrow$ $\frac{1}{5}$ = p

D-3. Head is obtained odd number of times = 1 head or 3 head or 5 head ....

P = ${}^nC_1 \left(\frac{1}{2}\right)^n + {}^nC_3 \left(\frac{1}{2}\right)^n + {}^nC_5 \left(\frac{1}{2}\right)^n$ + ... = $\left(\frac{1}{2}\right)^n$ {${}^nC_1 + {}^nC_3 + {}^nC_5$ + ...} = $\left(\frac{1}{2}\right)^n 2^{(n-1)} = \frac{1}{2}$

D-4. Draw of 2 coins can be done in the following ways



$\frac{{}^2C_2}{{}^5C_2} \times 2$ $\frac{{}^3C_2}{{}^5C_2} \times 4$ $\frac{{}^2C_1 \times {}^3C_1}{{}^5C_2} \times 3$

Value of expectation = $\frac{1}{{}^5C_2}$ (2 + 3 × 4 + 2 × 3 × 3) = 3.2

D-5. $E_A\left[\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+.....\right]\times 99 = 99\,\frac{\frac{1}{6}}{1-\frac{25}{36}} = 54$

D-6. $(q + p)^{99}$ $r \le \frac{99+1}{1+\left|\frac{1/2}{1/2}\right|}$ $\Rightarrow r \le \frac{100}{2}$ $\Rightarrow$ $r \le 50$

Terms 50 or 51 are highest, so r = 49, 50

PART - III

1. (A) Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

$\therefore$ probability = $\frac{2}{5}$

(B) $P(A \cap B) = \frac{1}{6}$ $\Rightarrow$ $P(A).P(B) = \frac{1}{6}$

$P(\bar{A}) = \frac{2}{3}$ $\Rightarrow$ $P(A) = \frac{1}{3}$

$\Rightarrow$ $P(B) = \frac{1}{2}$

$\therefore$ $6P\left(\frac{B}{\bar{A}}\right) = \frac{6P(B \cap \bar{A})}{P(\bar{A})} = \frac{6.P(B).P(\bar{A})}{P(\bar{A})} = 3$

(C) Total number of mapping = n^n. Number of one-one mapping = ${}^nC_1 . {}^{n-1}C_1$ .... ${}^1C_1$ = n!

Hence the probability = $\frac{n!}{n^n} = \frac{3}{32} = \frac{4!}{4^4}$. Comparing, we get n = 4.

(D) $625p^2 - 175p + 12 < 0$ gives $p \in \left(\frac{3}{25}\ ,\ \frac{4}{25}\right) \Rightarrow \left(\frac{4}{5}\right)^{n-1}.\frac{1}{5} = p$

$\therefore$ $\frac{3}{25} < \left(\frac{4}{5}\right)^{n-1}.\frac{1}{5} < \frac{4}{25}$

i.e. $\frac{3}{5} < \left(\frac{4}{5}\right)^{n-1} < \frac{4}{5}$ value of n is 3

2. (A) (6, 2), (2, 6), (3, 5), (5, 3), (4, 4)] $\to$ 5 ways

favourable = (3, 5) $\Rightarrow$ p = $\frac{1}{5}$

(B) A = 2 nd ball in white; B_1 = 1st ball in white; B_2 = 1st is black

$$P(B_1 / A) = \frac{p(A/B_1)p(B_1)}{p(A/B_1)p(B_1)+p(A/B_2)p(B_2)} = \frac{\frac{4}{7}\times\frac{3}{6}}{\frac{4}{7}\times\frac{3}{6}+\frac{3}{7}\times\frac{4}{6}}$$

(c) $\frac{2}{5} = (1 - P)\,P + (1 - P)^3\,P + (1\text{-}P)^5\,p + ...$; $\frac{2}{5} = P(1 - P)\{1 + (1 - P)^2 + (1 - P)^4 + ...\}$

$= P\ (1\text{-}P)\ \left(\frac{1}{1-(1-P)^2}\right)$ solving we get $p = \frac{1}{3}$

(D) (3,3,3,3) or (3,3,3,5) total $\to 2^4$. For = $1+\frac{4!}{3!} = 5 \Rightarrow p = \frac{5}{2^4}$

EXERCISE # 2

PART - I

1. Since line are more ${}^NC_M$ are those lines where telegrams will go ${}^NC_M \times M! =$ far

Total = N^M [As first telegram can go in any one of n lies]

[As 2nd telegram can go in any one of n lies] $P = \dfrac{{}^NC_M \;\; M\,!}{N^M}$

2. This problem is of conditional probability. Total cases in which at least one of the cubes is red painted is 125 – 27 = 98 out of which 8 are painted on three sides $\Rightarrow$ probability = $\dfrac{8}{98} = \dfrac{4}{49}$.

3. Since ten places are vacant. Probability of finding vacant places = $\dfrac{{}^{22}C_8}{{}^{24}C_{10}} = \dfrac{15}{92}$

4. A 3 $\left]\begin{matrix}3\text{ P}\\ 9\text{ B}\end{matrix}\right.$ B 2 $\left]\begin{matrix}3\text{ P}\\ 6\text{ B}\end{matrix}\right.$; P(A) = 1 – P(A) P(B) = 1 – P(B) = 1 – $\dfrac{{}^9C_3}{{}^{12}C_3}$ = 1 – $\dfrac{{}^6C_2}{{}^8C_2}$

5.

2							

The prime digits are (2, 3, 5, 7). If we fix 2 at first place, then other (2n – 1) places are filled by all four digits, so total number of cases = 4^{2n-1}

Now, sum of 2 consecutive digits is prime when consecutive digits are (2, 3) or (2, 5) then 2 will be fixed at all alternative places

2		2		2		2	

So favourable cases = 2^n. Therefore probability = $\dfrac{2^n}{4^{2n-1}} = 2^n.\, 2^{-4n+2} = 2^2.\, 2^{-3n} = \dfrac{4}{2^{3n}}$.

6. Clearly last 4 throws are same as first four $\Rightarrow$ probability = $\dfrac{2^4}{2^8} = \dfrac{1}{16}$

7. An urn contains 'm' green and 'n' red balls. K (< m, n) balls are drawn and laid aside, their colur being

$P(E_i) = \dfrac{{}^mC_i.\ {}^nC_{k-i}}{{}^{m+n}C_k}$; $P(A/E_i) = \dfrac{m-i}{m+n-k}$

$$P(A) = \sum_{j=0}^{k} \frac{{}^mC_j.{}^nC_{k-j}}{{}^{m+n}C_k} \times \frac{m-j}{(m+n-k)} = \sum_{j=0}^{k} \frac{m}{m-j}\ {}^{m-1}C_{m-j-1}{}^nC_{k-j} . \frac{m-j}{{}^{m+n}C_k(m+n-k)} \quad ......(1)$$

$$\sum_{j=0}^{k} {}^{m-1}C_j.\ {}^nC_{k-j} = \text{coeff. of } x^k \text{ in} (1+x)^{m+n-1} = {}^{m+n-1}C_k$$

$$\sum_{j=0}^{k} {}^{m-1}C_j.\ {}^nC_{k-j} = x^k (1+x)^{m+n-1} = {}^{m+n-1}C_k \quad .....(2)$$

Put (2) in (1) hence by solving $P(A) = \dfrac{m}{m+n}$

8. When 4 points are selected we get one intersecting point. So probability is $\dfrac{{}^nC_4}{{}^{({}^nC_2 - n)}C_2}$

Here, n = 10. So, probability is 6/17.

9. Let $w_1 \rightarrow$ ball drawn in the first draw is white, $b_1 \rightarrow$ ball drawn in the first drawn in black, $w_2 \rightarrow$ ball drawn in the second draw is white. Then $P(w_2) = P(w_1).\ P(w_2/\,w_1) + P(b_1)\ P(w_2/b_1)$

$$= \left(\frac{m}{m+n}\right)\left(\frac{m+k}{m+n+k}\right) + \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+k}\right) = \frac{m\ \ (m+k)+mn}{(m+n)(m+n+k)} = \frac{m\ \ (m+k+n)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

10. A ball from first urn can be drawn is two mannars

ball is white or ball is black

$P(w) = \dfrac{m}{m+n}$ $\qquad$ $P(B) = \dfrac{n}{m+n}$

Let E → selecting a white ball from second urn after a ball from urn first has been placed into it

$$P(E) = P(w)\ P(E/W) + P(B)\ P(E/B) = \frac{m}{m+n} \times \frac{p+1}{p+q+1} + \frac{n}{m+n}\ \frac{p}{p+q+1} = \frac{m(p+1)+np}{(m+n)(p+q+1)}$$

11. Given that '8' is 4th card. $E_1 \rightarrow$ '8' is of diamond $P(E_1) = 1/4$. $E_2 \rightarrow$ '8' is not of diamond $P(E_2) = 3/4$

Event 'A' :– top card is diamond. $P(A) = P(A/\in_1)\ P(A/\in_1) + P(A/\in_1)\ P(A/\in_1)$

$$= \left(\frac{12}{51}\right).\frac{1}{4} + \left(\frac{13}{51}\right) \times \frac{3}{4} = \frac{12 + 13 \times 3}{51 \times 4} = \frac{12 + 39}{51 \times 4} = \frac{1}{4}$$

12.

Case I	H	H	H	H	H				
Case II	T	H	H	H	H	H			
Case III		T	H	H	H	H	H		
Case IV			T	H	H	H	H	H	
Case V				T	H	H	H	H	H

Required probability $\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \frac{3}{32}$

13. A = coin tossed 5 times & falls head; B_1 = Both sided head coins

B_2 = one sided head coins; $p(B_1/A) = \dfrac{p(A/B_1)p(B_1)}{p(A/B_1)p(B_1)+p(A/B_2)p(B_2)} = \dfrac{\frac{1}{10}\times 1}{\frac{1}{10}\times 1 + \frac{9}{10}\times\left(\frac{1}{2}\right)^5}$

14. P(HA) = 0.8 ; P(HB) = 0.4. A = Only one bullet in bear.

B_1 = Shot by HA & missed by HB = $P(B_1) = 0.8 \times 0.60$

B_2 = Shot by HB & missed by HA = $P(B_2) = 0.4 \times 0.2$

$$P(B_1/A) = \frac{P(A/B_1)\ \ P(B_1)}{P(A/B_1)\ \ P(B_1)+P(A/B_2)\ \ P(B_2)} = \left(\frac{0.8\times 0.6}{0.8\times 0.6+0.2\times 0.4}\right) = \frac{48}{48+8} = 240$$

$E_A = 280 \times P(B_1/A)$ $\qquad$ $E_B = E - E_A$

15. P(Product of digits) = 12 $\qquad$ P = 12

if 34, 43, 26, 62 $\Rightarrow P(A) = \dfrac{4}{90} = \dfrac{2}{45} \Rightarrow P(\bar{A}) = \dfrac{43}{45}$ $\quad$ Probability $= 1 - \left(\dfrac{43}{45}\right)^3$

PART - II

1. $a_1 + a_2 + a_3 + ... + a_7 = 9k,\ k \in I$. Also $a_1 + a_2 + ... + a_9 = 1 + 2 + 5 + ... + 4 = 45$

$\Rightarrow \quad a_8 + a_9 = 45 - 9k \Rightarrow \quad 3 \le a_8 + a_9 \le 17$

$\Rightarrow \quad k = 4 \Rightarrow a_8 + a_9 = 9 \Rightarrow (1, 8)\ (2, 7), (3, 6), (4, 5)\ P(E) = \dfrac{4}{36} = \dfrac{1}{9}$.

2. n (S) = ways of selecting 3 number from 10 is ${}^{10}C_3$

n (E) → n $(A \cup B)$ where A → min. number chosen is 3 n (A) = ${}^{7}C_2$

B → max number chosen is 7

n (B) = ${}^{6}C_2$ also n $(A \cap B)$ = ${}^{3}C_1$ = 3 n (E) = ${}^{7}C_2 + {}^{6}C_2 - 3 \Rightarrow P = \dfrac{{}^{7}C_2 + {}^{6}C_2 - 3}{{}^{10}C_3}$

3. $P(C) = \dfrac{1}{{}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4} = \dfrac{1}{2^4 - 1} = \dfrac{1}{15}$; P(correct) = 1 – P (all wrong)

$= 1 - \dfrac{14}{15}\times\dfrac{13}{14}\times\dfrac{12}{13}\times\dfrac{11}{12}\times\dfrac{10}{11} = \dfrac{1}{3}$.

4. $P(x) = \dfrac{2}{3}$ $P(y) = \dfrac{3}{4}$ P(z) = ? P; P(2 bultets) $= \dfrac{11}{24}$

$\dfrac{11}{24} = \dfrac{2}{3}\times\dfrac{3}{4}(1 - P) + \dfrac{2}{3}\times\dfrac{1}{4}\times P + \dfrac{1}{3}\times\dfrac{3}{4}\times P$; $P = \dfrac{1}{2}$

5. A $\longrightarrow$ A

3 elements 3 elements

n(s) = number of mapping from A to A = 3^3 = 27

n(E) = number of one-one from A to A = 3! = 6

$P = \dfrac{6}{27} = \dfrac{2}{9}$

6. $\left(\dfrac{13}{52}\right)^2 \times \left(\dfrac{13}{52}\right)^2 \times \left(\dfrac{13}{52}\right)^2 \times \dfrac{6!}{2!\,2!\,2!} = P$

7. n(S) = 40! $\Rightarrow$ n(E) = 40!/3! $\Rightarrow P = \dfrac{1}{3!} = \dfrac{1}{6}$

Aliter n(S) = 40! $\Rightarrow$ n(E) = ${}^{40}C_3$. 1.37! $\Rightarrow$ $P = \dfrac{{}^{40}C_3 \times 37!}{40!} = \dfrac{1}{6}$

8. U1 – 1W + 1B U2 $\to$ 2W + 3B; U3 $\to$ 3W + 5B U4 $\to$ 4W + 7B

$P(W) = \sum_{i=1}^{4}(u_i)\ P(w/u_i) = \sum_{i=1}^{4}\dfrac{i^2+1}{34}\ P(w/v_i) = \dfrac{1^2+1}{34}\times\dfrac{1}{2} + \dfrac{2^2+1}{34}\times\dfrac{2}{5} + \dfrac{3^2+1}{34}\times\dfrac{3}{8} + \dfrac{4^2+1}{34}\times\dfrac{4}{11} = \dfrac{569}{1496}$

9. A = Letter drawn is vowel ; B_1 = written by Englishmen ; B_2 = written by American

$P(B_1/A) = \dfrac{P(A/B_1)\ P(B_1)}{P(A/B_1)\ P(B_1) + P(A/B_2)\ P(B_2)} = \dfrac{0.4\times\frac{3}{6}}{0.4\times\frac{3}{6} + 0.6\times\frac{2}{5}}$

10. P(identify high grade tea correctly) $= \dfrac{9}{10}$; P (identify low grade tea correctly) $= \dfrac{8}{10}$

P (Given high grade tea) $= \dfrac{3}{10}$; P (Given low grade tea) $= \dfrac{7}{10}$

P (Low grade tea / says high grade tea) $= \dfrac{\frac{7}{10}\times\frac{2}{10}}{\frac{7}{10}\times\frac{2}{10} + \frac{3}{10}\times\frac{9}{10}} = \dfrac{14}{41}$

11. Gambler might ruin in there ways :

(i) In first toss i.e T or (ii) In third toss i.e HTT or (iii) in fifith toss i.e HHTTT or HTHTT

$P = \dfrac{1}{2} + \dfrac{1}{8} + \dfrac{1}{32} + \dfrac{1}{32} = \dfrac{16+4+2}{32} = \dfrac{22}{32} = \dfrac{11}{16}$

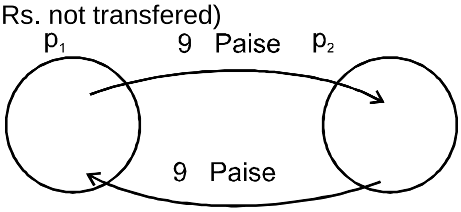
12. $A_1 \to$ red on both side $P(A_1) = \dfrac{1}{3}$; $A_2 \to$ red on upperside & blue on other $P(A_2) = \dfrac{1}{3}$

$A_3 \to$ blue on both side $P(A_3) = \dfrac{1}{3}$; E $\to$ cards showns upper side red

$P(E) = \dfrac{1}{3}\times 1 + \dfrac{1}{3}\times\dfrac{1}{2} + \dfrac{1}{3}\times 0 = \dfrac{1}{2}$; $P(A_1/E) = \dfrac{P(E/A_1)\ P(A_1)}{P(E)} = \dfrac{1\times\frac{1}{3}}{1/2} = \dfrac{2}{3}$

13. 10 coins 9 5 paisa 10 coins 5 paisa
1 1 Rs.

p = p (1 Rs. transfered + Back transfered) + p (1 Rs. not transfered)



$$\frac{{}^{9}c_8 \times {}^{1}c_1}{{}^{10}c_9} \times \frac{{}^{18}c_8 \times {}^{1}c_1}{{}^{19}c_9} + \frac{{}^{9}c_9 \times {}^{19}c_{19}}{{}^{10}c_9 \times {}^{19}c_{19}} = \frac{10}{19}$$

method 2 when 1 Rs coin is in second purse and did not came back in first purse this

prob. = $\frac{{}^{9}C_8 \times {}^{1}C_1}{{}^{10}C_9} \times \frac{{}^{18}c_9}{{}^{19}c_9} = \frac{9}{19} \Rightarrow$ Required probability = $1 - \frac{9}{19} = \frac{10}{19}$

14. "PARALLELOGRAM" or "PARALLELOPIPED" ⇒ A = RA is visible

B_1 = its from PARALLELOGRAM ⇒ B_2 = its from PARALLELOPIPED

$$P(B_1/A) = \frac{P(A/B_1)\ \ P(B_1)}{P(A/B_1)\ \ P(B_1) + P(A/B_2)\ \ P(B_2)} = \frac{\frac{1}{2}\times\frac{2}{12}}{\frac{1}{2}\times\frac{2}{12}+\frac{1}{2}\times\frac{1}{13}} = \frac{13}{19}$$

15. Unit digit of 3^a = 3, 9, 7, 1 each occurs 25 times in (0, 1, 2, ..., 99)

Unit digit of 7^b = 7, 9, 3, 1 each occurs 25 times in (0, 1, 2, ..., 99)

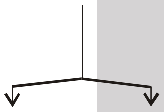
3^a	7^b
1	7

3^a	7^b
7	1

3^a	7^b
9	9

$$\Rightarrow \frac{25}{100}\times\frac{25}{100} + \frac{25}{100}\times\frac{25}{100} + \frac{25}{100}\times\frac{25}{100} = \frac{3}{16}$$

16. Bad brake = 0.3; $P(E_2)$ = not bad brake = 0.7 mechanic gives correct report $P(A_1)$ = 0.8. good brake come → bad brake given that mechanic says brakes are good



Bad brake Good brake
(0.3) × (0.2) (0.7) (1–0.2)

Probabilty that brakes are good = $\frac{0.7 \times 0.8}{0.3\times\ 0.2\ +\ 0.7\times0.8} = \frac{0.56}{0.06+\ 0.56} = \frac{0.56}{0.62} = \frac{28}{31}$

17. $\frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$ solve for n we get n = 5

18. M : Bolt is defective

B_1 : Produced by A; $P(B_1) = \frac{35}{100} = \frac{7}{20}$

B_2 : Produced by B; $P(B_2) = \frac{25}{100} = \frac{5}{20}$

B_3 : Produced by C; $P(B_3) = \frac{40}{100} = \frac{8}{20}$

$$P\left(\frac{B_3}{M}\right) = \frac{P(B_3)\ \ .\ \ P(M/B_3)}{\Sigma\ \ P(B_1)\ \ P(M/B_1)} = \frac{(.3)\left(\frac{8}{20}\right)}{(.2)\left(\frac{7}{20}\right)+(.1)\left(\frac{5}{20}\right)+(.3)\left(\frac{8}{20}\right)} = \frac{24}{14+5+24} = \frac{24}{43}$$

19.

First game	second game	third game	fourth game	fifth game
W	L	W	W	W
W	W	W	W	L
W	W	L	W	W
W	W	W	L	W
W	W	W	W	W

$P = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

$= \frac{4+8+4+4+16}{81} = \frac{36}{81} = \frac{4}{9}$

20. A : Exactly one children ; B : Exactly two children ; C : Exactly three children

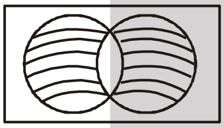
$P(A) = \frac{1}{4}$ $\quad P(B) = \frac{1}{2}$ $\quad P(C) = \frac{1}{4}$; E : Couple has exactly 4 grand children

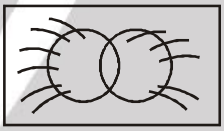
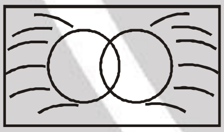
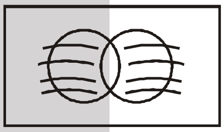
$P(E) = P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)$

$P(E) = \frac{1}{4}.0 + \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 + \frac{1}{4}.\frac{1}{4}.2\right] + \frac{1}{4}\left[3.\frac{1}{4}.\frac{1}{4}.\frac{1}{2}\right] = \frac{27}{128}$

PART - III

1. A = {1,3,5} ; B = {2,4,6}; C = {4,5,6}; D = {1,2}

2.  $P = P(M \cup N) - P(M \cap N) = P(M) + P(N) - 2P(M \cap N)$

(c) $P(\bar{M} \cup \bar{N}) - P(\bar{M} \cup \bar{N})$  –  = 

$\bar{M} \cup \bar{N}$ $\qquad$ $\bar{M} \cap \bar{N}$

3. A & B are independent $P(A \cup B)^c = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = P(\bar{A}) - P(B) + P(A)\,P(B)$

$= P(\bar{A}) - P(\bar{A})P(B) = P(\bar{A})P(\bar{B})$ and $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).\ P(B)}{P(B)} = P(A)$

4. E_1 : Both even or both odd $P(E_1) = \frac{^5C_2 + {^6C_2}}{^{11}C_2} = \frac{10+15}{55} = \frac{5}{11} \Rightarrow P(E_2) = 1 - P(E_1) = \frac{6}{11}$

(i) $P(E_1 / E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = 0 \Rightarrow P(E_2 / E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = 0$ (ii) E_1 and E_2 exhaustive

(iii) $P(E_2) > P(E_1)$

5. $P(A \cap B) = a$, $P(A) = a + d$, $P(B) = a + 2d \Rightarrow P(A \cup B) = a + 3d$ also $a + d = d$
$\Rightarrow a = 0 \Rightarrow P(A \cap B) = 0$, $P(A) = d$, $P(B) = 2d \Rightarrow P(A \cup B) = 3d$

6. $E_1 \longrightarrow$ p first digit is '2' $\Rightarrow$ 211 or 222 $\Rightarrow$ $\quad P(E_1) = \frac{2}{4} = \frac{1}{2}$

$E_2 \longrightarrow$ second digit is '2' $\Rightarrow$ 121, 222 $\Rightarrow$ $\quad P(E_2) = \frac{1}{2}$

$E_3 \longrightarrow$ third digit is '2' $\Rightarrow$ 222 & 112 $\Rightarrow$ $\quad P(E_3) = \frac{1}{2}$

$(E_1 \cap E_2) = 222 \Rightarrow P(E_1 \cap E_2) = \frac{1}{4} = P(E_1)\,P(E_2)$ Similiarly E_2 & E_3, E_1 & E_3

also $E_1 \cap E_2 \cap E_3 \to 222 \Rightarrow P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \neq P(E_1)\,P(E_2)\,P(E_3)$

7. $E_1 = \{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\}$

$E_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$; $E_3 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

(A) $E_1 \cap E_2 \cap E_3 = \phi$

(B) $E_1 \cap E_2 \neq \phi \Rightarrow E_2 \cap E_3 \neq \phi \Rightarrow E_1 \cap E_3 \neq \phi$

(C) $P(E_1 \cap E_2) = \frac{3}{36} = \frac{1}{12} \Rightarrow P(E_1).(E_2) = \frac{1}{24} \Rightarrow P(E_1 \cap E_2) \neq P(E_1)\,P(E_2)$

E_1E_2 are not independent.

(D) $P\left(\frac{E_3}{E_1}\right) = \frac{P(E_3 \cap E_1)}{P(E_1)} = \frac{1/36}{1/4} = \frac{2}{9}$

8. (A) $(1 - 0.1)^4$

(B) P(more than 3) = P(all four) = $(0.1)^4$

(C) P(not more than 3) = 1 – P(more than 3) = $1 - (0.1)^4$

(D) P(all four) = $(0.1)^4$

9. At end of any number there could be 10 possible digits n (S) = 10 × 10 × 10 × 10 to get last digit of product 1, 3, 7 or 9, end place should be occupid by these digits only. Hence n (E) = 4^4 $P = \frac{4^4}{10^4}$

Probability that the last digit in the product is 0 is $\frac{10^4 - 8^n - 5^n + 4^n}{10^n} = \frac{5535}{10^4}$

10. Last place can be occupied by (0 – 9) 10 methods.

to get '6' at unit place of x^4 Last digit should be 2, 4 ,6 or 8 is 4 ways $\Rightarrow P = \frac{4}{10} \equiv 40\%$

11. $\frac{np}{npq} = \frac{3}{2} \Rightarrow q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$

$r \leq \frac{11}{1+\frac{2}{3}} \Rightarrow r \leq \frac{10}{3} \Rightarrow r \leq 3.33$

thus 3 success is most probable.

12. $P(T_1) = p \Rightarrow P(T_2) = q \Rightarrow P(T_3) = 1/2$

$= P(T_1, T_2, T_3) + P(T_1, T_2, T_3) + P(T_1, T_2, T_3) \Rightarrow \frac{1}{2} = pq(1/2) + p(1 - q)\frac{1}{2} + pq\frac{1}{2}$

$\frac{1}{2} = \frac{pq}{2} + \frac{p}{2} \Rightarrow 1 = pq + p$. Now, check options.

13. $P(E_0) = \frac{3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)}{3!} = \frac{1}{3}$; $P(E_1) = \frac{{}^3C_1(1).2!\left(1 - \frac{1}{1!} + \frac{1}{2!}\right)}{3!} = \frac{1}{2}$

$P(E_2) = \frac{{}^3C_2(1)^2.1!\left(1 - \frac{1}{1!}\right)}{3!} = 0$; $P(E_3) = \frac{{}^3C_3(1)^3}{3\,!} = \frac{1}{6}$

$P(E_0) + P(E_3) = P(E_1) \Rightarrow P(E_0).\,P(E_1) = P(E_3)$

$\because$ $E_0 \cap E_1 = \phi$

$P(E_0 \cap E_1) = 0$ $P(E_0 \cap E_1) = P(E_2)$

14. (A) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) + P(B) - P(A \cup B)}{P(B)} \geq \frac{P(A) + P(B) - 1}{P(B)}$

(B) $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$ $\therefore$ A & B are ind.

$= P(A)\,(1 - P(B)) + P(B) = P(A)\,P(\bar{B}) + P(B) + 1 - 1 = P(A)\,P(\bar{B}) - P(\bar{B}) + 1$

$\Rightarrow 1 + P(\bar{B})\,(P(A) - 1) = 1 - P(\bar{A})\,P(\bar{B})$

15. $p(x=4) = {}^{n}c_4\left(\frac{1}{2}\right)^n$
$p(x=5) = {}^{n}c_5\left(\frac{1}{2}\right)^n$
$p(x=6) = {}^{n}c_6\left(\frac{1}{2}\right)^n$
$\Rightarrow 2\ {}^{n}c_5 = {}^{n}c_4 + {}^{n}c_6 \Rightarrow 4\ {}^{n}c_5 = {}^{n+1}c_5 + {}^{n+1}c_6 \Rightarrow 4\ {}^{n}c_5 = {}^{n+2}c_6$

$4.\ \frac{n!}{5!\ (n-5)!} = \frac{(n+2)!}{6\ \ !(n-4)!} \Rightarrow 4 = \frac{(n+2)(n+1)}{6(n-4)} \Rightarrow 24\ (n\text{-}4) = (n+2)\ (b+1) \Rightarrow\ n = 7, 14$

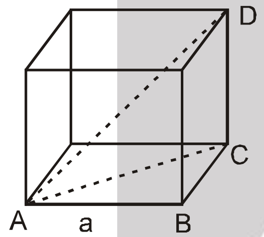
16. $\sum_{x=0}^{\infty} P(X = x) = 1 \Rightarrow x = \frac{16}{25}$

17. $X = \{a_1, a_2, \text{..........}a_n\}$; number of subset of $X = 2^n$; ways of choosing A & B = 2^{2n}
ways of choosing A & B so that they have same number of element is
${}^{n}C_0.{}^{n}C_0 + {}^{n}C_1.{}^{n}C_1 + \text{.....} + {}^{n}C_n.{}^{n}C_n = {}^{2n}C_n$

18. Total ways = 3^9
(A) : Favourable cases = 3^6
(B) : Favourable cases = 3^3
(C) : Favourable cases = 3^6
(D) : Favourable cases = 3^6

PART - IV

1.



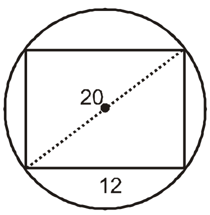
Diagonal A D will be diameter of sphere. $AD^2 = AC^2 + DC^2 = a^2 + a^2 + a^2 = 3a^2 \Rightarrow AD = \sqrt{3}\ a$

Volume of sphere is $\frac{4}{3}\pi\left(\frac{\sqrt{3}.a}{2}\right)^3$ Volume of cube = a^3

Eequired probability = $1 - \frac{\text{volume of cube}}{\text{volume of sphere}} = 1 - \frac{a^3}{\pi\frac{\sqrt{3}}{2}a^3} = 1 - \frac{2}{\pi\sqrt{3}}$

2. $20^2 - 12^2 = x^2 \Rightarrow x^2 = 400 - 144$

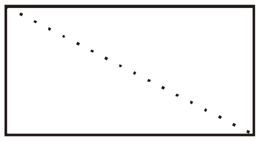
$x = \sqrt{256} = 16$; $P = \frac{16\times 12}{\pi\ \ 10^2}$



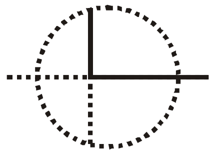
3. $0 < x < 10$ $\quad x^2 + y^2 < 100$

$0 < y < 10$

$p = \dfrac{\text{चतुर्थांश का क्षेत्रफल}}{\text{आयत का क्षेत्रफल}} = \dfrac{\frac{1}{4}\Pi \times 10^2}{10 \times 10}$ x



y



(Q 4 & 6)

Sol. P (studies 10 hrs per day) = 0.1 = $P(B_1)$; P (studies 7 hrs per day) = 0.2 = $P(B_2)$

P (studies 4 hrs per day) = 0.7 = $P(B_3)$; A : successful

4. $P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) = \frac{1}{10} \times \frac{80}{100} + \frac{2}{10} \times \frac{60}{100} + \frac{7}{10} \times \frac{40}{100} = \frac{48}{100} = \frac{12}{25}$

5. $P(B_3/A) = \dfrac{P(B_3).P(A/B_3)}{\Sigma P(B_1).P(A/B_1)} = \dfrac{\frac{7}{10} \times \frac{40}{100}}{\frac{12}{25}} = \dfrac{7}{12}$

6. $P(B_3/\overline{A}) = \dfrac{P(B_3).P(\overline{A}/B_3)}{\Sigma P(B_1).P(\overline{A}/B_1)} = \dfrac{\frac{7}{10} \times \frac{60}{100}}{\frac{1}{10} \times \frac{20}{100} + \frac{2}{10} \times \frac{40}{100} + \frac{7}{10} \times \frac{60}{100}} = \dfrac{420}{520} = \dfrac{21}{26}$

Sol. (Q 7 & 8) A : Person draw 2 white and 2 Red; B : Person draw 3 White and 1 Red,
C : Person draw 4 White; E : 4 ball are drawn in which atleast 2 are white

$P(E) = P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right) + P(C).P\left(\frac{E}{C}\right)$; $P(E) = \frac{1}{3}\left[\frac{{}^4C_2.{}^6C_2}{{}^{10}C_4} + \frac{{}^4C_3.{}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4}{{}^{10}C_4}\right]$

$P\left(\frac{A}{E}\right) = \frac{90}{115}$ $P\left(\frac{B}{E}\right) = \frac{24}{115}$ $P\left(\frac{C}{E}\right) = \frac{1}{115}$

E_1 = A ball is drawn again and found to be white $P(E_1) = \frac{90}{115} \times \frac{2}{6} + \frac{24}{115} \times \frac{1}{6} + \frac{1}{115} \times 0 = \frac{34}{115}$

(9) Identical letters are 1,1,2,2,3,3. Total words = $\frac{8!}{2!\ 2!\ 2!} = 5040$

Number of words in which all identical letters are together = 5!

Number of words in which only exactly two pair of identical digits appear together = $\left(\frac{6!}{2!} - 5!\right) \times 3 = 720$

Number of words in which only one pair of identical digits appear together (1, 1) together

$= \left(\frac{7!}{2!\ 2!} - \{240 + 240 + 120\}\right) \times 3 = 1980$ Now number of words in which no two identical digits appear together = 5040 – (1980 + 720 + 120) = 2220 Probability = $\frac{2220}{5040} = \frac{37}{84}$

(10) Number of words in which only exactly two pair of identical digits appear together = $\left(\frac{6!}{2!} - 5!\right) \times 3 = 720$

Total words = $\frac{8!}{2!\ 2!\ 2!} = 5040$; Probability = $\frac{720}{5040} = \frac{1}{7}$.

EXERCISE # 3

PART - I

1. $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$; r_1, r_2, r_3 are to be selected from {1, 2, 3, 4, 5, 6}. As we know that $1 + \omega + \omega^2 = 0$

$\therefore$ from r_1, r_2, r_3, one has remainder 1, other has remainder 2 and third has remainder 0 when divided by 3.

$\therefore$ we have to select r_1, r_2, r_3 from (1, 4) or (2, 5) or (3, 6) which can be done in ${}^2C_1 \times {}^2C_1 \times {}^2C_1$ ways value of r_1, r_2, r_3 can be interchanged in 3! ways.

$\therefore$ required probability = $\dfrac{({}^2C_1 \times {}^2C_1 \times {}^2C_1) \times 3!}{6 \times 6 \times 6} = \dfrac{2}{9}$

2. Probability (P) = $\dfrac{P(GGG) + P(GRG)}{P(GGG) + P(GRG) + P(RGG) + P(RRG)}$

$$\Rightarrow P = \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}} \Rightarrow P = \frac{36+4}{36+4+3+3} = \frac{40}{46} = \frac{20}{23}$$

3. $P(\text{white}) = P(H \cap \text{white}) + P(T \cap \text{white}) = \frac{1}{2}.\left\{\frac{3}{5}\times 1 + \frac{2}{5}.\frac{1}{2}\right\} + \frac{1}{2}.\left\{\frac{{}^3C_2}{{}^5C_2}\times 1 + \frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3} + \frac{{}^3C_1.{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$

$$= \frac{1}{2}\times\frac{8}{10} + \frac{1}{2}\times\left\{\frac{3}{10} + \frac{1}{30} + \frac{12}{30}\right\} = \frac{4}{10} + \frac{1}{2}\times\frac{22}{30} = \frac{23}{30}$$

4. $$P\left(\text{Head}/\text{White}\right) = \frac{P(\text{Head} \cap \text{white})}{P(\text{white})} = \frac{\frac{1}{2}\times\left\{\frac{3}{5}\times 1 + \frac{2}{5}\times\frac{1}{2}\right\}}{\frac{23}{30}} = \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$$

5.* $P(E \cap F) = P(E) . P(F)$ ....(1)

$P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$ ....(2)

$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$ ....(3)

by (2) $P(F) + P(E) - 2P(E \cap F) = \frac{11}{25}$ ....(4)

by (3) $1 - [P(E) + P(F) - P(E \cap F)] = \frac{2}{25}$ $[P(E) + P(F) - P(E \cap F)] = \frac{23}{25}$ ....(5)

by (4) & (5) $P(E)\,P(F) = \frac{12}{25}$ ....(6)

and $P(E) + P(F) = \frac{7}{5}$ ....(7)

By (6) and (7) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ or $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

6*. $P(x_1) = \frac{1}{2}$; $P(x_2) = \frac{1}{4}$; $P(x_3) = \frac{1}{4}$

$P(x) = P(E_1 E_2 E_3) + P(\bar{E}_1 E_2 E_3) + P(E_1 \bar{E}_2 E_3) + P(E_1 E_2 \bar{E}_3) = \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{1}{4} + \frac{1}{2}.\frac{3}{4}.\frac{1}{4} + \frac{1}{2}.\frac{1}{4}.\frac{3}{4}$

$P(x) = \frac{1}{4}$ $\Rightarrow$ (A) $P\left(\frac{x_1^c}{x}\right) = \dfrac{P(x_1^c \cap x)}{P(x)} = \dfrac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$

(B) P(exactly two / x) = $\dfrac{P(\text{exactly two} \cap x)}{P(x)} = \dfrac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \dfrac{7}{8}$

(C) P(x / x_2) = $\dfrac{P(x\cap x_2)}{P(x_2)} = \dfrac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{4}} = \dfrac{5}{8}$

(D) P(x / x_1) = $\dfrac{P(x\cap x_1)}{P(x_1)} = \dfrac{\frac{1}{2}.\frac{1}{4}.\frac{1}{4}+\frac{1}{2}.\frac{3}{4}.\frac{1}{4}+\frac{1}{2}.\frac{1}{4}.\frac{3}{4}}{\frac{1}{2}} = \dfrac{7}{16}$

7. Favourable : D_4 shows a number and
only 1 of $D_1D_2D_3$ shows same number
or only 2 of $D_1D_2D_3$ shows same number
or all 3 of $D_1D_2D_3$ shows same number

8*. $P(X/Y) = \frac{1}{2} \Rightarrow \dfrac{P(X\cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3} \Rightarrow P(Y/X) = \frac{1}{3} \Rightarrow \dfrac{P(X\cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$ A is correct

$P(X \cap Y) = P(X) \cdot P(Y) \Rightarrow$ X and Y are independent B is correct

$P(X^c \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ D is not correct

9. P (problem solved by at least one) = 1 – P(problem is not solved by by all)

$= 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})\,P(\bar{D}) = 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$

10. Let x, y, z be probability of E_1, E_2, E_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha \quad \Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma \quad \Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get x = 2y and y = 3z $\Rightarrow$ x = 6z $\Rightarrow \frac{x}{z} = 6$

11.

Bag 1	Bag 2	Bag 3
1 W	2 W	3 W
3 R	3 R	4 R
2 B	4 B	5 B

$\Rightarrow$ P(W W W) + P(R R R) + P(B B B)

$\left(\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}\right) + \left(\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}\right) + \left(\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}\right) \Rightarrow \dfrac{6+36+40}{6\times 9\times 12} \Rightarrow \dfrac{82}{648}$

12. P (Ball drawn from box 2 / one is W one is R) = $\dfrac{P(A\cap B)}{P(B)}$

$= \dfrac{P(A\cap B)}{P(B)} = \dfrac{\frac{1}{3}\times\frac{2\times 3}{^9C_2}}{\frac{1}{3}\left[\frac{1\times 3}{^6C_2}+\frac{2\times 3}{^9C_2}+\frac{3\times 4}{^{12}C_2}\right]} = \dfrac{\frac{2\times 3\times 2}{9\times 8}}{\frac{3\times 2}{6\times 5}+\frac{2\times 6\times 2}{9\times 8}+\frac{3\times 4\times 2}{12\times 11}}$

$= \dfrac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \dfrac{\frac{1}{6}}{\frac{66+55+60}{55\times 60}} = \dfrac{55}{181}$

13. 3 Boys & 2 Girls..................
(1) B (2) B (3) B (4)
Girl can't occupy 4^{th} position. Either girls can occupy 2 of 1, 2, 3 position or they can both be a position (1) or (2).
Hence total number of ways in which girls can be seated is ${}^3C_2 \times 2! \times 3! + {}^2C_1 \times 2! \times 3! = 36 + 24 = 60$.
Number of ways in which 3 B & 2 A can be seated = 5 !
Hence required prob. = $\frac{60}{5!} = \frac{1}{2}$.

14. $x_1 + x_2 + x_3$ is odd if all three are odd or 2 are even & one is odd
$$\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7} + \frac{2}{3}\times\frac{2}{5}\times\frac{3}{7} + \frac{1}{3}\times\frac{3}{5}\times\frac{3}{7} + \frac{1}{3}\times\frac{2}{5}\times\frac{4}{7} = \frac{24+12+9+8}{105} = \frac{53}{105}$$

15. $2x_2 = x_1 + x_3$.
If x_1 & x_3 both are odd $2 \times 4 = 8$ ways
x_1 & x_3 both are even $1 \times 3 = 3$ ways
Total = 11 ways
Total $(x_1\ x_2\ x_3)$ triplets are $3 \times 5 \times 7 \Rightarrow P = \frac{11}{105}$

16. Let coin is tossed n times
P(atleast two heads) = $1 - \left(\frac{1}{2}\right)^n - {}^nC_2.\left(\frac{1}{2}\right)^n \geq 0.96 \Rightarrow \frac{4}{100} \geq \frac{n+1}{2^n}$
$\Rightarrow \quad \frac{n+1}{2^n} \leq \frac{1}{25} \quad \Rightarrow \quad \frac{2^n}{n+1} \geq 25 \Rightarrow$ least value of n is 8.

17. $\text{Box}-\text{I} < \begin{matrix}\text{Red}\to n_1\\ \text{Black}\to n_2\end{matrix}$ $\text{Box}-\text{II} < \begin{matrix}\text{Red}\to n_3\\ \text{Black}\to n_4\end{matrix}$ $P(R) = \frac{1}{2}.\frac{n_1}{n_1+n_2} + \frac{1}{2}.\frac{n_3}{n_3+n_4}$

$$R(II/R) = \frac{\frac{1}{2}.\frac{n_3}{n_3+n_4}}{\frac{1}{2}.\frac{n_1}{n_1+n_2}+\frac{1}{2}.\frac{n_3}{n_3+n_4}} = \frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}$$
by option $n_1 = 3,\ n_2 = 3,\ n_3 = 5,\ n_4 = 15$
$$P(II/R) = \frac{\frac{5}{20}}{\frac{3}{6}+\frac{5}{20}} = \frac{n_4}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{4}\times\frac{4}{2+1} = \frac{1}{3}$$

18. Given $\frac{n_1}{n_1+n_2}.\frac{n_1-1}{n_1+n_2-1} + \frac{n_2}{n_1+n_2}.\frac{n_1}{n_1+n_2-1} = \frac{1}{3}$
$3(n_1^2 - n_1 + n_1n_2) = (n_1 + n_2)(n_1 + n_2 - 1)$
$3n_1(n_1 + n_2 - 1) = n_1 + n_2(n_1 + n_2 - 1)$
$2n_1 = n_2$

19. Let x = P(computer turns out to be defective given that it is produced in Plant T_2),
$\frac{7}{100} = \frac{1}{5}\times(10x) + \frac{4}{5}x \quad \Rightarrow \quad 7 = 200x + 80x \quad \Rightarrow \quad x = \frac{7}{280}$
P(produced in T_2/ not defective) = $\frac{P(A\cap B)}{P(B)}$
$$\frac{4/5(1-x)}{\frac{1}{5}(1-10x)+\frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right)+\frac{4}{5}\left(\frac{273}{280}\right)} = \frac{4\times 273}{210+4\times 273} = \frac{2\times 273}{105+2\times 273} = \frac{546}{651} = \frac{78}{93}$$

$$\frac{4/5(1-x)}{\frac{1}{5}(1-10x)+\frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right)+\frac{4}{5}\left(\frac{273}{280}\right)} = \frac{4\times 273}{210+4\times 273} = \frac{2\times 273}{105+2\times 273} = \frac{546}{651} = \frac{78}{93}$$

20. $P(X > Y) = T_1T_1 + DT_1 + T_1D$ (Where T_1 represents wins and D represents draw)

$\frac{1}{2}.\frac{1}{2}+\frac{1}{6}.\frac{1}{2}+\frac{1}{2}.\frac{1}{6}=\frac{5}{12}=$

$\Rightarrow$ (B) is correct

21. $P(X = Y) = DD + T_1T_2 + T_2T_1 = \frac{1}{6}.\frac{1}{6}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{2} = \frac{1}{36}+\frac{1}{3} = \frac{39}{36\times 3} = \frac{13}{36}$ $\Rightarrow$ (C) is correct

22. $\frac{P(X\cap Y)}{P(Y)} = \frac{1}{2}$ $\frac{P(Y\cap X)}{P(X)} = \frac{2}{5}$ $P(X\cap Y) = \frac{P(Y)}{2} = \frac{2}{5}P(X) = \frac{2}{5}.\frac{1}{3} = \frac{2}{15} \Rightarrow P(Y) = \frac{4}{15}$

$$\frac{P(\bar{X}\cap Y)}{P(Y)} = \frac{P(Y)-P(X\cap Y)}{P(Y)} = \frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}} = \frac{2}{4} = \frac{1}{2}$$

$P(X\cup Y) = P(X) + P(Y) - P(X\cap Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

23. $x + y + z = 10$

Total number of non-negative solutions = ${}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Now Let $z = 2n$. $x + y + 2n = 10$; $n \ge 0$

Total number of non-negative solutions = 11 + 9 + 7 + 5 + 3 + 1 = 36

Required probability = $\frac{36}{66} = \frac{6}{11}$

24. Probability = $\frac{4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{5!} = \frac{9}{120} = \frac{3}{40}$

25. Total cases = 5!

favorable ways =14

$\underline{1}\ \underline{3}\ \underline{5}\ \underline{2}\ \underline{4}$
$\underline{1}\ \underline{4}\ \underline{2}\ \underline{5}\ \underline{3}$ } $\rightarrow$ 2 5 $\rightarrow$ 2

$\underline{2}\ \underline{4}\ \underline{1}$ ... ...$\rightarrow$ 2
4 $\rightarrow$ 3
$\underline{2}\ \underline{5}\ \underline{3}\ \underline{1}\ \underline{4}\rightarrow 1$

$\underline{3}\ \underline{1}\ \underline{5}\ \underline{2}\ \underline{4}$
$\underline{3}\ \underline{1}\ \underline{4}\ \underline{2}\ \underline{5}$ } $\rightarrow$ 2 $\underline{3}$ $\underline{5}$ ... ...} $\rightarrow$ = 14

Probability = $\frac{14}{120}$

26.

	Bag_1	Bag_2	Bag_3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

(A) $P(\text{Ball is Green}) = P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3) = \frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8} = \frac{39}{80}$

(B) P(Ball chosen is Green/ Ball is from 3rd Bag) = $\frac{3}{8}$

(CD) P(Ball is from 3rd Bag / Ball chosen is Green) = $\frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3)}$

$$P(B_1) = \frac{3}{10} \Rightarrow P(B_2) = \frac{3}{10} \Rightarrow P(B_3) = \frac{4}{10} = \frac{\frac{4}{10}\times\frac{3}{8}}{\frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}} = \frac{4}{13}$$

27. E_2 : Sum of elements of A = 7 $\Rightarrow$ These are 7 ones and 2 zeros. Number of such matrices = ${}^9C_2 = 36$.
Out of all such matrices; E_1 will be those when both zeros lie in the same row or in the same colume

eg. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $n(E_1 \cap E_2) = 2 \times {}^3C_2 \times {}^3C_2 = 18$,

$\uparrow \quad \uparrow$

So n(E1/E2) = $\frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2}$

28. A and B are independent events P(A) P(B) = P(A $\cap$ B) $\Rightarrow \frac{a}{6}\times\frac{b}{6} = \frac{c}{6} \Rightarrow ab = 6c$ |A| = a,|B| = b,|A $\cap$ B|=c

(a, b, c) =(3, 2, 1) so ${}^6C_1{}^5C_2\,{}^3C_1 = 180$
=(4, 3, 2) so ${}^6C_2\,{}^4C_2\,{}^2C_1 = 180$
=(6, 1, 1) so ${}^6C_1 = 6$
=(6, 2, 2) so ${}^6C_2 = 15$
=(6, 3, 3) so ${}^6C_3 = 20$
=(6, 4, 4) so ${}^6C_4 = 15$
=(6, 5, 5) so ${}^6C_5 = 6$ Total = 360 + 62 = 422

PART - II

1. Statement-1 Total ways = ${}^{20}C_4$
number of AP's of common difference 1 is = 17
number of AP's of common difference 2 is = 14
number of AP's of common difference 3 is = 11
number of AP's of common difference 4 is = 8
number of AP's of common difference 5 is = 5
number of AP's of common difference 6 is = 2

total = 57

probability= $\frac{57}{{}^{20}C_4} = \frac{1}{85}$ Statement-2 common difference can be ± 6, so statement-2 is false. Hence correct option is (2)

2. $= \frac{{}^3C_1\,{}^4C_1\,{}^2C_1}{{}^9C_3} = \frac{\frac{3\,.\,4\,.\,2}{9\,.\,8\,.\,7}}{3\,.\,2\,.\,1} = \frac{2}{7}$. Hence correct option is (1).

3. $1 - P^5 \geq \frac{31}{32}$; $P^5 \leq \frac{1}{32}$; $P \leq \frac{1}{2}$; $P \in \left[0, \frac{1}{2}\right]$

4. $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$; $\geq 1 \frac{1}{P(D)}$; $\frac{P(C)}{P(D)} \geq P(C)$; $P(C) \leq P\left(\frac{C}{D}\right)$

5. $P(A^c \cap B^c/C) = \frac{P((A^c \cap B^c) \cap C)}{P(C)} = \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)} = 1 - P(A) - P(B) = P(A^c) - P(B)$

6. Let Event (Given : {1, 2, 3,.........8})
A : Maximum of three numbers is 6.
B : Minimum of three numbers is 3

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

7. $p = \frac{1}{3}$, $q = \frac{2}{3}$; ${}^5C_4 \left(\frac{1}{3}\right)^4 . \frac{2}{3} + {}^5C_5 \left(\frac{1}{3}\right)^5 = 5. \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

8. Given $P\left(\overline{AUB}\right) = \frac{1}{6}$, $P\ (A \cap B) = \frac{1}{4}$, $P\left(\overline{A}\right) = \frac{1}{4}$

$\therefore 1 - P(A \cup B) = \frac{1}{6} \Rightarrow 1 - P(A) - P(B) + P(A \cup B) = \frac{1}{6} \Rightarrow 1 - \frac{3}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$ ($\therefore$ P(A) = 1 – P ($\overline{A}$))

$\Rightarrow$ $P(B) = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$

A and B are not equally likely. Further P(A). P(B) = $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A \cap B)$ A and B are independent events

9. There seems to be ambiguity in the question. It should be mentained that boxs are different and one particular box has 3 balls : then number of ways = $\frac{{}^{12}C_3 \times 2^9}{3^{12}}$ = $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Alter :

${}^3C_1\ {}^{12}C_3\ ({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_4 + {}^9C_5 + {}^9C_5 + {}^9C_7 + {}^9C_8 + {}^9C_9)$

$+ \frac{\lfloor 12 \times \lfloor 3}{\lfloor 3 \lfloor 3 \lfloor 6 \lfloor 3}$ $= {}^3C_1\ {}^{12}C_3\ (2^9 - 2\,{}^9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}$

correct answer should have been $\frac{{}^3C_1\,{}^{12}C_3(2^9 - 2.{}^9C_3) + \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 6}}{3^{12}}$

10. E_1 : {(4, 1) ,............... (4,6,)} 6 cases

E_2 : {(1,2), ... (6,2)} 6 cases

E_3 : 18 cases (sum of both are odd)}

$P(E_1) = \frac{6}{36} = \frac{1}{6} = P\ (E_2) \Rightarrow P(E_3) = \frac{18}{36} = \frac{1}{2} \Rightarrow P\ (E_1 \cap E_2) = \frac{1}{36} \Rightarrow P(E_2 \cap E_3) = \frac{1}{12}$

$P(E_3 \cap E_1) = \frac{1}{12}$

$P(E_1 \cap E_2 \cap E_3) = 0$

$\therefore E_1, E_2, E_3$ are not independent

11. $P(A) + P(B) - 2P\ (A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$P\ (A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) = \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

12. $P = \frac{6}{{}^{11}C_2} = \frac{6}{55}$

$x_1 - x_2 = \pm 4\lambda$

$x_1 + x_2 = 4\alpha$

$2x_1 = 4\ (\lambda \pm \alpha)$

$x_1 = 2\ (\lambda \pm \alpha)$

x_1	x_2
0	4, 8

2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

13. 15 green + 10 yellow = 25 balls

P(green) = $\frac{3}{5} = p_1$

P(yellow) = $\frac{2}{5} = q$

n =10

$\therefore$ Variance = npq = $10.\ \frac{3}{5}.\frac{2}{5} = \frac{60}{25} = \frac{12}{5}$

14. 4R + 6B = 10

$p = \frac{4}{10}.\frac{6}{12} + \frac{6}{10}.\frac{4}{12} \qquad = \frac{24}{120} + \frac{24}{120} = \frac{2}{5}$

15. $P(x = 1) = \frac{4}{52} \times \frac{48}{52} \times 2 = \frac{24}{169}$

$P(x = 2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} \quad \Rightarrow \quad P(x = 1) + P(x = 2) = \frac{25}{169}$

16. Sum of all elements of S is 210 Let x denotes a nice set

then x could be S –{7} , S –{1 , 6}, S –{2, 5}, S –{3, 4}, S –{1, 2, 4} hence required probability is $\frac{5}{2^{20}}$

17. P(_ _ _ 44) = P(4 _ _ 44) + P(not4 _ _ 44)

$= \frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{5}{6}\times 1\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6} = \frac{25}{6^5}+\frac{25}{6^4} = \frac{175}{6^5}$

18. p(success) = p(5 or 6) = $\frac{1}{3}$ expectations equal to 100/3 + 100/9 – 400/9 = 0

Aliter : In each thrown expectation of gaining rupees = $\frac{2}{3}(-50) + \frac{1}{3}(100) = 0$

$\Rightarrow$ Therefor expectation is zero

19. $A \subset B$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq P(A)$ (always) सदैव

$P(A/B) = \frac{P(A)}{P(B)} \geq P(A)$

20. $1 - \frac{1}{2^n} > \frac{9}{10} \Rightarrow \frac{1}{10} > \frac{1}{2^n} \Rightarrow 2^n > 10 \qquad \therefore$ minimum value of n is 4

21.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = no. of times head occur consecutively
Now expectation

$= \sum xP(k) = (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32} + 3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32} = \frac{1}{8}$

22. Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27\times17}{64\times8}$

$= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$

$\therefore k = \frac{17}{8}$

23. AA + ABA + BAA + ABBA + BBAA + BABA $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

HIGH LEVEL PROBLEMS (HLP)

1. Let $A_1 \rightarrow$ Ball drawn from urn A is red and ball returned is also red, $P(A_1) = \frac{6}{10} \times \frac{5}{11}$

$B_1 \rightarrow$ Ball drawn from urn A is red but ball returned to it is black, $P(B_1) = \frac{6}{10} \times \frac{6}{11}$

$C_1 \rightarrow$ Ball drawn from urn A is black and ball of same colour is returned, $P(C_1) = \frac{4}{10} \times \frac{7}{11}$

$D_1 \rightarrow$ Ball drawn from urn A is black and ball returned is red, $P(D_1) = \frac{4}{10} \times \frac{4}{11}$

Required probability $P(R) = P(A_1) \times P\left(\frac{R}{A_1}\right) + P(B_1) \times P\left(\frac{R}{B_1}\right) + P(C_1) \times P\left(\frac{R}{C_1}\right) + P(D_1) \times P\left(\frac{R}{D_1}\right)$

$= \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} + \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} + \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} + \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{32}{55}$

2. P = P(1 person dies) + P(2 person dies) P(A 1 dies ; first / 2 person died) + P(3 person died)

P(A 1 died first / 3 person died) = $^nC_1 \; pq^{n-1} \times \frac{1}{n} + ^nC_1 \; p^2q^{n-2} \times \frac{1}{2} + ^nC_2 \; p^3q^{n-3} \times \frac{1}{3} + ...$

$= pq^{n-1} + ^{n-1}C_{r-1} \; p^2q^{n-2} \frac{1}{2} + ^nC_3 \; p^3q^{n-3} \frac{1}{3} + ... = pq^{n-1} + \sum_{r=2} {}^{n-1}C_{r-1} \; p^r \; q^{n-r} \frac{1}{r}$

As $\frac{^{n-1}C_{r-1}}{r} = \frac{^nC_1}{n}$; $P = Pq^{n-1} + \frac{1}{n}\sum_{r=2}^{n} {}^nC_r P^r q^{n-r} \Rightarrow P = pq^{n-1} + \frac{1}{n}(1 - ^nC_0 \; p^0q^n - ^nC_1 \; P^1q^{n-1})$

$P = Pq^{n-1} + \frac{1}{n}(1 - q^n - nPq^{n-1}) = \frac{1-(1-p)^n}{n}$

3. Three squares are shown as below

I^{st} row	1								
II^{nd} row		2							
III^{rd} row				3					

$\frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} = \frac{2}{21}$

digit 3 may come only in I^{st} and II^{nd} rows. In second square if? is replaced by 3 then probability is 1/3.

Case-1 : We assume that first square contains digit 3 in first row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in II^{nd} row $\therefore$ probability is 3/6

Case-2 : We assume that first square contains digit 3 in second row $\therefore$ probability is 2/7

and corresponding to it in third square digit 3 may come in I^{st} row $\therefore$ probability is 3/6

Hence probability $= \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{2}{7} \times \frac{3}{6} = \frac{2}{21}$

4. n(S) = ways of sitting of 10 boys and 5 girls = 15 !

S_1..............S_a.................S_b................S_c...............S_5

x y z w

Girl Girl Girl Girl Girl

Let end seats are occupied by the girls & between first and second girl x boys are seated similarly between second and third y boys

.... so on then x + y + z + w = 10 where x, y, z, w are (2k + 1) type $\Rightarrow 2k_1 + 1 + 2k_2 + 1 + 2k_3 + 1 + 2k_4 + 1 = 10$

$\Rightarrow \quad k_1 + k_2 + k_3 + k_4 = 3 \qquad$ where $k_i \geq 0$

number of solution are ${}^{3+4-1}C_{4-1} = {}^6C_3 \Rightarrow$ n (E) = ${}^6C_3 \times 10! \times 5! \Rightarrow$ Now P = $\dfrac{{}^6C_3 \times 10! \times 5!}{15!}$

5. Probability of same no. of wins and losses = no wins no losses + 1 win, 1 loss + 2 wins, 2 loss

$$= \left(\frac{1}{3}\right)^5 + {}^5C_2.2\left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)^3 + {}^5C_2.{}^3C_2.\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^5 + (1+20+30) = \frac{17}{81}$$

$\Rightarrow$ Probability that A wins more matches than its losses = $\dfrac{1}{2}\left(1-\dfrac{17}{81}\right) = \dfrac{32}{81}$

6. Total no. of +ve integral solutions of x + y + z + w = 21 is ${}^{21-1}C_{4-1}$ = 1140. Let n be the no. of solutions in which x > y. then n be the solutions in which x < y and m be the solutions in which x = y. we must have 2n + m = 1140. Now, if x = y. then the equation is 2x + z + w = 21

If x = 1, z + w = 19 has 18 solutions

If x = 2, z + w = 17 has 16 solutions

⋮ ⋮ ⋮

If x = 9, z + w = 3 has 2 solutions

$\therefore \quad m = 18 + 16 + ... + 2 = 2 \times \dfrac{9 \times 10}{2} = 90 \Rightarrow 2n + 90 = 1140 \Rightarrow n = 525$

Desired probability = $\dfrac{525}{1140} = \dfrac{35}{76}$

7. Probability of getting all red faces in throws by die P = $\dfrac{1}{2}.\left(\dfrac{4}{6}\right)^n$. Probability of getting all red faces i-

throws by die Q = $\dfrac{1}{2}.\left(\dfrac{2}{6}\right)^n$ Probability that die P was being used = $\dfrac{\frac{1}{2}\left(\frac{4}{6}\right)^n}{\frac{1}{2}\left(\frac{4}{6}\right)^n + \frac{1}{2}\left(\frac{2}{6}\right)^n} = \dfrac{2^n}{2^n+1}$

8. total no. of possible = 6! ; favorable cases = ${}^6C_2.4!.\left[1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}\right]$ = 15 × 9

Desind probability = $\dfrac{15 \times 9}{6!} = \dfrac{3}{16}$

9. Clearly $p_1 = p_2 = 1$, $p_3 = \dfrac{215}{216}$

1 2 3 ....n – 3 n – 2 n – 1 n

.... $\bar{1}$ → $p_{n-1} \cdot \frac{5}{6}$

.... $\bar{1}$ 1 → $p_{n-2} \cdot \frac{5}{6} \cdot \frac{1}{6}$

.... $\bar{1}$ 1 1 → $p_{n-3} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$

$$p_n = \frac{5}{6}.\frac{1}{6}.\frac{1}{6}.p_{n-3} + \frac{5}{6}.\frac{1}{6}.p_{n-2} + \frac{5}{6}.p_{n-1}$$

10. Let E_r = event that exactly r students do not appear. Then P (E_r) = kr

So, P (E_1) + P (E_2) + ……… + P (E_n) = 1 $\Rightarrow$ k(1 + 2 + ….. + n) = 1 $\Rightarrow k = \dfrac{2}{n(n+1)}$

Let A_j = event that exactly j students are selected out of n– r

Then P (A_j/E_r) = $k_r j$ So, $1.k_r + 2k_r + \ldots\ldots\ldots + (n-r)k_r = 1 \Rightarrow k_r = \dfrac{2}{(n-r)(n-r+1)}$

Let B = event that exactly two students are selected. Then P(B)=P(E_{n-2}) P($A_2/(E_{n-2})$) + P(E_{n-3}) P($A_2/(E_{n-3})$)+…….+P(E_1) P(A_2/E_1)+P(E_0)P (A_2/E_0) = $k(n-2).2k_{n-2} + k(n-3).2k_{n-3} + \ldots\ldots + k.1.2k_1$

$$= 4k\left(\frac{n-2}{2\cdot 3}+\frac{n-3}{3\cdot 4}+.......+\frac{n-(n-1)}{n(n-1)}\right) = 4K\left[n\left(\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+.......+\frac{1}{n(n-1)}\right)-\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+.......+\frac{1}{n}\right)\right]$$

$$= \frac{8}{n(n+1)}\left[n\left(\frac{1}{2}-\frac{1}{n}\right)-\left(\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n}\right)\right]$$

11. Let p and q denote probability of things going to man and woman respectively.

Therefore $p = \frac{1}{1+\mu}$ and $q = \frac{\mu}{1+\mu}$

Probability of men receiving r things is given by

$P_r = {}^nC_r .\ q^{n-r}.p^r.$

So required probability is given by $P_1 + P_3 + P_5 + ...$

$$= \frac{1}{2}\left[(q+p)^n-(q-p)^n\right] = \frac{1}{2}\left[1-\left(\frac{\mu-1}{\mu+1}\right)^n\right] = \frac{1}{2}-\frac{1}{2}\left(\frac{\mu-1}{\mu+1}\right)^n$$

By comparison, we have $\left(\frac{\mu-1}{\mu+1}\right) = \frac{1}{2} \Rightarrow 2\mu - 2 = \mu + 1$. Thus $\mu = 3$. **[1]**

12. (i) Since Smith's sister has blue eyes both his parents must have a blue eyed gene.

P(both of Smith's parenets has a blue eyed gene) = 1

(ii) Since his parents has brown eyes their gene pair is brown-blue.

Smith's possibilities → {Br–Br, Br-Bl, Bl–Br}

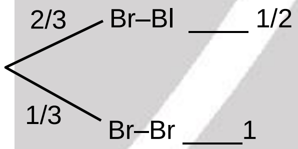
(As Smith's has brown eyes (Bl–Bl) is not possible)

P(Smith has blue eyed gene) = $\frac{2}{3}$

(iii) As Smith's wife has blue eyes both her genes are blue so she will donate blue gene to the progeny

P(Smith donates blue eyed gene) = $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$ = P(Smith's first child has blue eyes)

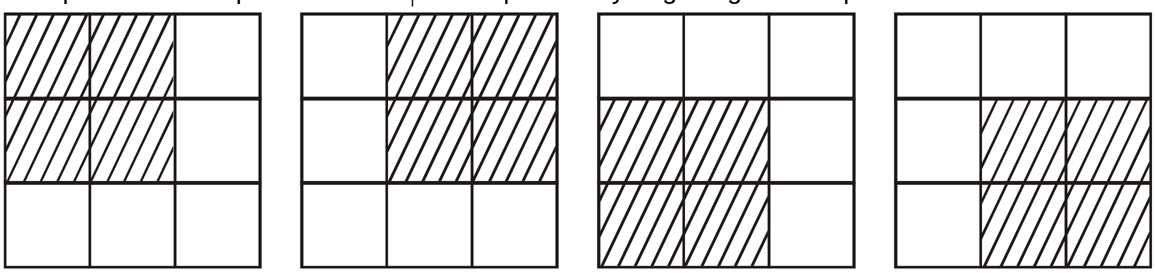
(iv)



P(both gene brown/child has brown eyes) = $\dfrac{\frac{1}{3}\times 1}{\frac{2}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{1}{2}$

(v) The probability of the child having brown eyes is 2/3

13. Four possible red square are Let P_i be the probability of getting i^{th} red square



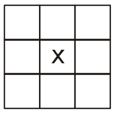
(1) (2) (3) (4)

$\Rightarrow$ $P_1 = P_2 = P_3 = P_4 = \frac{1}{16}$ and $P_{12} = P_{13} = P_{24} = P_{34} = \frac{1}{64}$ (Inclusive exclusion principle)

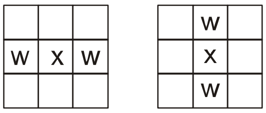
$P_{14} = P_{23} = \dfrac{1}{128}$ and $P_{1234} = \dfrac{1}{1512}$

$\Rightarrow$ P(no. of red square) $= 1 - \dfrac{1}{4} + \dfrac{5}{64} - \dfrac{1}{64} + \dfrac{1}{512} = \dfrac{417}{512}$

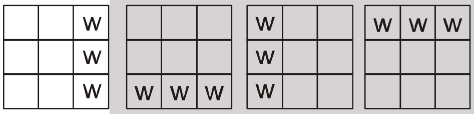
Alitir : No. white, all red = 1 ; one white, 8 red = 8



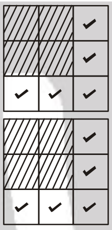
two white, 7 red = $^8C_2 - 2$



three white, 6 red = $(^5C_2 - 2) \times 4 + 4$



or



$(^5C_2 - 2) \times 4$

fourth white, 5 red = 5 × 4 five white, 4 red = 4

$1 + 8 + (^8C_2 - 2) + \{(^5C_2 - 2) \times 4 + 4\} + 5 \times 4 + 4$

$= 9 + 26 + 32 + 4 + 20 + 4 = 30 + 52 + 13 = 95 = \dfrac{512 - 95}{512} = \dfrac{417}{512}$.

14. If series of at least m heads starts from first throw = 2^{m+1} no. of ways. Similarly if it starts for i^{th} throw (i = 2, 3, ..., m + 2) = 2^m a case when first m times and last m times are heads have be counted twice.

So total ways are = $2^{m+1} + 2^m(m + 1) - 1$. Required probability = $\dfrac{(m+3)2^m - 1}{2^{2m+1}}$

15. n (S) = 6 × 6 × 6 = 216 also to get sum of '8' $x_1 + x_2 + x_3 = 8$ where $1 \le x_i \le 6$

$x_i = t_i + 1$ $\quad 0 \le t_1 \le 5 \Rightarrow t_1 + t_2 + t_3 = 5$

by fictious partition method number of solution of this equation is $^7C_2$

n (E) = 21 $\Rightarrow$ $P = \dfrac{21}{216} = \dfrac{7}{72}$

16. If A takes r shot than B will take more than r shots so

required probabilities = $\sum P(A')^{r-1} . P(A) [P(B')^r . P(B) + P(B')^{r+1}. P(B) + ...]$

$= \sum (P(A'))^{r-1}. P(A). \dfrac{(P(B'))^r P(B)}{1 - P(B')} = \sum_{r=1}^{\infty} (P(A'))^{r-1} .P(A) . (P(B'))^r = \dfrac{P(A).P(B')}{1 - P(A').P(B')} = \dfrac{3/5.2/7}{1 - 2/5.2/7} = \dfrac{6}{31}$

17. Let quadratic equation is $ax^2 + bx + c = 0$

Since $\alpha + \beta = \alpha^2 + \beta^2$ & $\alpha\beta = \alpha^2\beta^2$ $\Rightarrow$ $\alpha\beta = 0$ or $\alpha\beta = 1 \Rightarrow \alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$

If $\alpha = 0, \beta = \beta^2$ $\Rightarrow$ $\beta = 0$ or 1 $\Rightarrow$ roots are (0,0) (0,1)

If $\beta = 0$ $\alpha = \alpha^2$ $\Rightarrow$ $\alpha = 0$ or 1 $\Rightarrow$ roots are (0,0) (1,0)

When $\alpha\beta = 1$ $\quad \alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow (\alpha + \beta) = (\alpha + \beta)^2 - 2$

$\Rightarrow (\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0 \Rightarrow \alpha + \beta = 2$ or $\alpha + \beta = -1$

When $\alpha + \beta = 2$ we get $\alpha = \beta = 1$. When $\alpha + \beta = -1$ we get $\alpha + \dfrac{1}{\alpha} = -1$ give imaginary roots

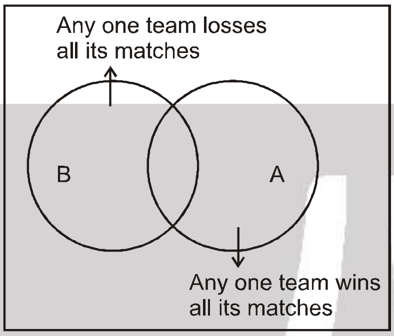
$\Rightarrow$ roots are (0,0) (1,0) (0,1) (1,1) $\Rightarrow P = \dfrac{2}{4} = \dfrac{1}{2}$

18. P(there is a team winning all its matches) = P(A) = $^5C_1 . \left(\frac{1}{2}\right)^4$ P(there is a team losing all its matches)

= P(B) = $^5C_1 . \left(\frac{1}{2}\right)^4$ P(a team is winning all its matches and other team is losing all its matches)

= $P(A \cap B) = 2.\ ^5C_2 . \left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^3$ P(No team is winning all its matches or lossing all its matches)

$= 1 - P(A \cap B) = 1 - \{P(A) + P(B) - P(A \cap B)\} = \frac{17}{32}$



19. Suppose that A and B each toss and coins. Let E_{ij} denote the even that A gets i heads and B gets j heads. We have $P(E_{ij}) = \left(^nC_i . \frac{1}{2^n}\right)\left(^nC_j . \frac{1}{2^n}\right) = \frac{^nC_i . ^nC_j}{2^{2n}}$

E_1 denote the event that A gets more head than B and E_2 the event that A and B get the same no. of heads. We have $E_1 = \bigcup_{i>j} E_{ij}$ & $E_2 = \bigcup_{i} E_{ii} \Rightarrow P(E_1) = \sum_{i>j} P(E_{ij}) = \sum_{i>j} \frac{^nC_i . ^nC_j}{2^{2n}}$

$P(E_2) = \sum_i P(E_{ii}) = \sum_{i=0}^{n} \frac{(^nC_i)^2}{2^{2n}} = 2^{2n} \Rightarrow \sum_{i>j} {^nC_i} . {^nC_j} = \frac{2^{2n} - {^{2x}C_n}}{2}$

$P(E_1) = \frac{2^{2n} - {^{2n}C_n}}{2^{2n+1}}$ & $P(E_2) = \frac{^{2n}C_n}{2^{2n}}$

Let E denote the even that A gets more heads than B when A tosses (n + 1) coins and tosses n coins. If E_1 has already occurred, then the out come of the $(n + 1)^{th}$ toss is immatereal. If E_2 has already occured than n the outcoms of $(n + 1)^{th}$ coin must be a head.

$P(E) = P(E_1)\ P(H \text{ or } T/E_1) + P(E_2) : P(H/E_2) = \frac{2^{2n} - {^{2n}C_n}}{2^{2n+1}}.1 + \frac{^{2n}C_n}{2^{2n}}.\frac{1}{2} = \frac{2^{2n} - {^{2n}C_n} + {^{2n}C_n}}{2^{2n+1}} = \frac{1}{2}$

20. Let E_j denote the event that the number of children in the family is j. Let A denote the event that the family has exactly k boys. We have

$P(E_j) = \alpha p^j$ (j = 0, 1, ...) and $P(A/E_j) = \begin{cases} {^jC_k}(1/2)^j & j \geq k \\ 0 & j < k \end{cases}$

$P(A) = \sum_{j=0}^{\infty} P(E_j)\, P(A/E_j) = \sum_{j=k}^{\infty} \alpha p^j . {^jC_k} \left(\frac{1}{2}\right)^j = \alpha \sum_{r=0}^{\infty} {^{k+r}C_k} . \left(\frac{p}{2}\right)^{k+r} = \alpha \left(\frac{p}{2}\right)^k \sum_{r=0}^{\infty} {^{k+r}C_r} \left(\frac{p}{2}\right)^r$

we know that |x| < 1 and a +ve integer m $(1 - x)^{-m} = 1 + {^mC_1}\, x + {^{m+1}C_2}\, x^2 + ...$

$\sum_{r=0}^{\infty} {^{k+r}C_k} . \left(\frac{p}{2}\right)^r = \left(1 - \frac{p}{2}\right)^{-k-1} \Rightarrow P(A) = \frac{\alpha (p/2)^k}{\left(1 - \frac{p}{2}\right)^{k+1}} = \frac{2\alpha\, 2^k}{(2-p)^{k+1}}$

21. Let T denotes the event that the bear is hit when x bullets are fired at bush A.

Let E_1, E_2 denots the event as $P(E_1)$ =; $P(E_2)$ $\frac{9}{25} = \frac{16}{25}$.

so $P(T/E1) = 1 - (3/4)^x$ and $P(T/E_2) = 1 - (3/4)^{10-x}$

Now $P(x) = {}^5C_x \left(\frac{1}{2}\right)^5 \left[\frac{9}{25}\left(1-\left(\frac{3}{4}\right)^x\right)+\frac{16}{25}\left(1-\left(\frac{3}{4}\right)\right)^{5-x}\right]$

Now put x = 1, 2, 3, 4, 5 in p(x) and find out the maximum p(x). for x = 1, 2 we get maximum value of p(x)

22. Event (1) : selection of Set
A : Selection of set A
B : Selection of set B
event (2) : Selecting a number corresponding to a year
L. Y. : selecting a number correspond to leap year
S.Y. : selecting a number correspond to simple year
event (3) : number of sundays in selected year. 53S : Selecting year has 53 sundays.

$P(53S) = P(A).\ P\left(\frac{L.Y}{A}\right)\ P\left(\frac{53S}{L.Y}\right) + P(A)\ .\ P\left(\frac{S.Y}{A}\right)\ P\left(\frac{53S}{S.Y}\right) + P(B)\ .$

$P\left(\frac{L.Y}{B}\right) P\left(\frac{53S}{L.Y}\right) + P(B)\ .\ P\left(\frac{S.\quad Y}{B}\right)\ .\ P\left(\frac{53S}{S.Y}\right)$

$= \frac{1}{2}\cdot\frac{24}{100}\cdot\frac{2}{7}+\frac{1}{2}\cdot\frac{76}{100}\cdot\frac{1}{7}+\frac{1}{2}\cdot\frac{25}{100}\cdot\frac{2}{7}+\frac{1}{2}\cdot\frac{75}{100}\cdot\frac{1}{7} = \frac{249}{1400}$

Probability that the chosen year was a leap year $= \dfrac{\frac{1}{2}.\frac{24}{100}\times\frac{2}{7}+\frac{1}{2}\times\frac{25}{100}\times\frac{2}{7}}{\frac{249}{1400}} = \frac{98}{249}$

23. Let probability of success in a trial is 'p'. Then $P(X = r) = {}^{10}C_r\, p^r (1-p)^{10-r}$

Given that at r = 4 we achive maximum value of P(X = r)

$r = \dfrac{10+1}{1+\frac{1-p}{p}} = 11p \Rightarrow [r] = 4 \Rightarrow 4 < r < 5 \Rightarrow 4 < 11p < 5 \Rightarrow \frac{4}{11} < p < \frac{5}{11}$

24. Probability of any article is defective from 1st lot $= \frac{n}{N}$

Probability of any article is defective from 2nd lot $= \frac{m}{M}$

Hence probability that an article selected at random from the new lot is defective

$= \dfrac{\frac{n}{N}.K+\frac{m}{M}.L}{K+L} = \dfrac{KnM + LmN}{MN(K+L)}$

25. $x_1 + x_2 + x_3 = 10$ where $1 \le x_i \le 6$ coefficient of x^{10} in $(x^1 + x^2 + ... + x^6)^3$
= coefficient of x^7 in $(1 + x^1 + x^2 + ... + x^5)^3$
= coefficient of x^7 in $(1 - x^6)^3 \sum {}^{3+r-1}C_r\, x^r$ = coefficient of x^7 in $(1 - 3x^6) \sum {}^{3+r-1}C_r\, x^r = {}^9C_2 - 3.\ {}^3C_1 = 27$

Aliter : n (S) = 6 × 6 × 6 n(E) is number of solution of $x_1 + x_2 + x_3 = 10$ where $1 \le x_1, x_2, x_3 \le 6$

$x_i = t_i + 1 \qquad 0 \le t_i \le 5$

$t_1 + t_2 + t_3 = 7$

Hence ${}^9C_2$ – any one get more than 5 $= {}^9C_2 - {}^3C_1 \times {}^3C_2 = 27$. Required probability $= \frac{27}{6\times6\times6} = \frac{1}{8}$

26. A may win in following manner

(i) W	W	W	
(ii) W	W	L	W
(iii) W	L	W	W
(iv) L	W	W	W

$P(A) = \frac{1}{8} + {}^3C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{8} + \frac{3}{16} = \frac{5}{16} \Rightarrow P(B) = \frac{11}{16}$

$\Rightarrow$ expectation of A is $\frac{5}{16} \times 1600$ = Rs 500 $\Rightarrow$ expectation of B is $\frac{11}{16} \times 1600$ = Rs 1100

27. P (A) = P (C) clearly undoward ; A $\to$ no boy or exactly one boy in family

$\Rightarrow \quad P(A) = \left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

B $\to$ 2 boy , 1 girl or 1 boy , 2 girl ; $P(B) = {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 = \frac{6}{8} = \frac{3}{4}$

C : no girl or exactly one girl ; $P(C) = \left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 = \frac{1}{2}$

$A \cap B \to$ one boy & two girls ; $P(A \cap B) = {}^3C_1\left(\frac{1}{2}\right)^3 = 3/8$

$B \cap C \to$ one girl and two boys $P(B \cap C) = \frac{3}{8}$

$A \cap C \to \phi \quad \Rightarrow A \cap B \cap C$ in also ϕ. $P(A \cap B) = \frac{3}{8} = P(A) \times P(B)$, so A and B are independent

neither $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$ nor $P(A \cap C) = P(A) \times P(C)$. So ABC are not independent

28. Let a segment of line is x, then other is (a – x), where 0 < x < a
Since no part is greater than b $\Rightarrow$ x < b and a – x < b $\Rightarrow$ x > a – b

or a – b < x < b Now $P = \frac{\text{favorable length}}{\text{total length}} = \frac{2b-a}{a}$

29. Total ways $2^8 \times 2^8$. Favourable ways $({}^{16}C_8) = \frac{{}^{16}C_8}{2^{16}}$

(Q. 30 to 32) Total numbers of ways of selecting r_1, r_2, r_3, r_4 is 8^4

30. For y = 4 $\Rightarrow$ r_1, r_2, r_3, r_4 can have values equal to 4 or 8 i.e $2^4 = 16$

31. for y = –4 $\Rightarrow$ r_1, r_2, r_3, r_4 can have values equal to 2 or 6 i.e $2^4 = 16$

32. for y = 0 following cases are possible

(i) $r_1 = 1, 5$ $r_2 = 2, 4,$ $r_3 = 3, 7$ $r_4 = 4, 8$

$2^4 \times 4!$

(ii) $r_1 = 1, 5$ $r_2 = 3, 7,$ $r_3 = 1, 5$ $r_4 = 3, 7$

$\frac{2^4 \times 4!}{2!\ \ 2!}$

or

(iii) $r_1 = 2, 5$ $r_2 = 4, 8,$ $r_3 = 2, 6$ $r_4 = 4, 8$

$\frac{9}{64}$

33. $P(E_i) = ki(i+1) \Rightarrow \quad k\sum_{i=1}^{n} i(i+1) = 1 \quad \Rightarrow k = \frac{3}{n(n+1)(n+2)}$

$P(E_n) = \frac{3}{n(n+1)(n+2)} \times n\ (n+1) = \frac{3}{n+2}$

$P(E) = \sum_{i=1}^{n} P(E_i).P\left(\frac{E}{E_0}\right) = \sum_{i=1}^{n} ki\ (i+1)\ .\frac{i}{n} = \frac{(3n+1)(n+2)}{4n(n+2)} = \frac{3n+1}{4n}$

$P(E_1/E) = \frac{P(E_1).P(E/E_1)}{P(E)} = \frac{K.2.\frac{1}{n}}{\frac{(3n+1)\ \ (n+2)}{4n(n+2)}} = \frac{24}{n(n+1)(n+2)(3n+1)}$